

## NEW ZEALAND SCHOLARSHIP 2004

### STATISTICS AND MODELLING

#### Sample of assessed candidate work – Performance Descriptor 2: Performance Category 5

Note: This was a high level of performance by this candidate who missed achieving Performance Descriptor 1 overall through an inability to critically evaluate processes and solutions (no AC's awarded). Refer to the assessment schedule for marking codes.

#### QUESTION ONE

(a) Find a 95% confidence interval for the percentage support of all customers for this proposal.

$$\mu = \frac{46}{200} = 0.23$$

$$\sigma = \sqrt{\frac{0.23 \times (1 - 0.23)}{200}} = 0.0298$$

$$e = z\sigma = 1.96 \times \sigma = 0.0583$$

$$\therefore \mu + e = 0.2883 = 28.83\%$$

$$\mu - e = 0.1717 = 17.17\%$$

$$\therefore 95\% \text{ confidence interval is } 17.17\% < x < 28.83\%$$

BP

(b) Assuming that the percentage support of all customers is the value given by the lower limit of the interval in part (a), calculate the probability that the level of support from another sample of 200 customers is at least 25%.

This situation has a binomial distribution

$$\sigma = \sqrt{n\pi(1-\pi)} \\ = 5.333$$

$$\mu = 0.1717 \times 200 = 34.34$$

$$x = 200 \times 25\% = 50$$

Use normal distribution approximation:

$$Z = \frac{49.5 - 34.34}{5.333} = 2.843 \text{ (continuity correction)}$$

$$\therefore P = 0.4977$$

$$0.5 - 0.4977 = 0.0023$$

$$\therefore \text{Probability for level of support exceeds 25\% is } 0.0023$$

BP

- (c) NAILS claims that at least 30% of all customers support this proposal. Can this claim be justified? Give a reason for your answer.

When  $\chi = 200 \times 30\% = 60$

$$Z = \frac{60 - 34.34}{5.333} = 4.812 > 4$$

Therefore the probability for it to occur is approximately 0.  
This claim is therefore not justified.

BS

- (d) The manager of NAILS requests that a second survey be conducted so that the width of the interval in (a) is reduced by 40%. How many customers should be interviewed in the second survey?

$$e^1 = (1-40\%)e = 60\%e$$

$$z^1 \sigma^1 = 60\% z \sigma \quad z^1 = z = 1.96$$

$$\therefore \sigma^1 = 60\% \sigma$$

$$\sqrt{\frac{\pi(1-\pi)}{n^1}} = 60\% \sqrt{\frac{\pi(1-\pi)}{n}}$$

$$\frac{\pi(1-\pi)}{n^1} = 0.36 \times \frac{\pi(1-\pi)}{n}$$

$$\therefore 0.36n^1 = n \quad \therefore n^1 = 2.778n$$

$$0.36n^1 = n \quad n = 200$$

$$\therefore n^1 = 555.5 \approx 556$$

556 of customers should be interviewed.

BP

## QUESTION TWO

- (i) Find the third equation

For azalea  $\frac{1}{3}a$  of Nitrogen (N), phosphorus (P) and potassium (K) are used

For bulb  $\frac{4}{15}b$  of N,  $\frac{5}{15}b$  of P and  $\frac{6}{15}b$  of K are used.

For all-purpose  $\frac{5}{18}c$  of N,  $\frac{6}{18}c$  of P and  $\frac{7}{18}c$  of K are used.

$$\therefore \frac{1}{3}a + \frac{4}{15}b + \frac{5}{18}c = 5.3 \quad (1)$$

$$\frac{1}{3}a + \frac{5}{15}b + \frac{6}{18}c = 8.4 \quad (2)$$

$$\frac{1}{3}a + \frac{6}{15}b + \frac{7}{18}c = 11.5 \quad (3)$$

Rearrange these equations

$$(1) \times 90 \Rightarrow 30a + 24b + 25c = 477 \quad (4)$$

(first equation)

$$(2) \times 90 \Rightarrow 30a + 30b + 30c = 756 \quad (5)$$

(second equation)

$$(3) \times 90 \Rightarrow 30a + 36b + 35c = 1035 \quad (6)$$

$\therefore$  third equation is  $30a + 36b + 35c = 1035$

BM

(ii) Find the general solution for this system of equations in terms of  $c$ .

$$(5) - (4) \quad 6b + 5c = 279$$

$$(6) - (5) \quad 6b + 5c = 279$$

$$\therefore b = \frac{279 - 5c}{6}$$

Substitute  $b = \frac{279 - 5c}{6}$  into equation (4)

$$30a + 1116 - 20c + 25c = 477$$

$$30a = -639 - 5c \quad a = \frac{-639 - 5c}{30}$$

BM

(iii) Comment on the practical consequences of the solution in part (ii).

Practically  $a \geq 0$ ,  $b \geq 0$ ,  $c \geq 0$

$$\text{However if } c \geq 0 \quad a = \frac{-639 - 5c}{30} \leq 0$$

This contradicts with the condition  $a \geq 0$  above.

The equations would result in either  $a$  or  $c$  (maybe  $b$ ) with a negative value. Hence it is not usable practically.

NS

In order to achieve an AC for Q2(iii), the candidate needed to comment on the practical implications of their answer; for example, the fact they will have a wastage and/or shortfall of N, P and K. If they wanted to overcome this, they would need to order more.

### QUESTION THREE

Write a report (approximately one page long) to the general manager of NAILS that summarises the statistical output given below. Include some sales predictions in your report.

General manager:

From the data collected during the time period of Jan 2001 to Dec 2002, the relationship between the promotional expenditure and the total sales can be concluded as:

Overall, higher expenditure is, higher the sale is, increase in expenditure has its significant effect in the increase of sales.

However, different value of expenditure leads to different increase of the sale.

When the expenditure is between 0 to 15 thousands of dollars, the sales increase reasonably significantly as expenditure increases.

When the expenditure is between 15 and 30 thousands, the sale increases the most sharply with gradient of 3.67 (about twice as much as the 1.51 gradient for the whole graph).

However the increase slows down again when expenditure exceeds 30 thousands. At 45 thousands or above, the increase in sales as expenditure increases is minute and insignificant.

When expenditure exceeds 60 thousands, it almost has no effect to sales.

Expenditure of 30 thousand is recommended because when E exceeds 30 thousand, the gradient of graph is smaller than 1, i.e. increase in sale is less than the increase of expenditure put in. The outliers are excluded from the analysis because they are random variation.

BS

NS

## QUESTION FOUR

- (a) The management of NAILS wants to know how many bottles of both POW and ZAP should be produced. The preliminary estimates of their potential profitability are \$20 per bottle of POW and \$10 per bottle of ZAP. Perform an appropriate analysis and make a recommendation to management about the amount of POW and ZAP that could be produced to maximize profit.

If  $\chi$  bottles of POW and  $y$  bottles of ZAP are produced:

$$30\chi + 20y \leq 1500 \quad y \leq -1.5\chi + 75$$

$$\chi \leq 35$$

$$\chi + y \leq 65 \quad y \leq -\chi + 65$$

$$\chi \geq 15, y \geq 15$$

These constraints are drawn in graph at top of page

$$P = 20\chi + 10y$$

$$\therefore y = -2\chi + \frac{P}{10}$$

The function intercept with C with the highest  $y$  intercept at the same time i.e. C has the maximum profit.

$$\text{At point C, } \chi = 35 \quad y = -1.5 \times 35 + 75 \\ = 22.5 \approx 22$$

$$P = 35 \times 20 + 10 \times 22 = 920$$

Therefore it is recommended to produce 35 bottles of POW and 22 bottles of ZAP to have the potential max profit of \$920.

[NB. Candidate's graph omitted.]

NS

Insufficient evidence provided to meet the requirements. The candidate has only provided one of the two solutions required.

- (b) Suppose that the profit of \$20 per bottle of POW was an overestimate and it was in reality only \$15 per bottle. How does that change your production recommendation in part (a)?

When profit of X is now \$15

$$P = 15\chi + 10y$$

$$y = -1.5\chi + \frac{P}{10}$$

And now the function overlaps with line  $y = -1.5\chi + 75$  when it reaches the highest y intercept.

Therefore any point along the line BC between point B, C would give maximum profit at Point B

$$-\chi + 65 = -1.5\chi + 75$$

$$\therefore \chi = 20 \text{ and } y = 45$$

Therefore the new max profit can be obtained when  $\chi$  bottle of POW are produced while  $20 \leq \chi \leq 35$  and  $\chi \leq z$  and y bottle of ZAP is produced while  $y = -1.5\chi + 75$ ,  $y \leq z$

$$P = 15 \times 20 + 10 \times 45 \\ = 750$$

Now this max profit is \$750.

AT

- (c) Suppose x is the number of bottles of POW that should be produced to meet management's request and maximise the profit. Find an implicit equation for x.

$$y = -1.5\chi + 75 \quad y = 9 \ln \chi$$

$$-1.5\chi + 75 = 9 \ln \chi$$

AT

- (d) Solve the equation derived in part (c) and make a recommendation to management about the amount of POW and ZAP that could be produced to maximize profit.

$$f(\chi) = -9 \ln \chi - 1.5\chi + 75$$

$$f'(\chi) = -\frac{9}{\chi} - 1.5$$

Using Newton Raphson method

$$\text{With } \chi_0 = 30$$

$$\chi = 29.661$$

$$\approx 30$$

$$\text{When } \chi = 30$$

$$y = 30.5 \approx 30$$

$\therefore$  now maximum profit is obtained when 30 bottles of each ZAP and POW are sold.

BM

QUESTION FIVE

(a) Two percent of punnets from one supplier, *Grow Well*, are known to be defective.

- Two truckloads of punnets from *Grow Well* arrive at NAILS. Calculate the probability that exactly one truckload of the two is accepted.

First truck

Second truck

Binomial distribution  $\pi = 0.02$   
 $n = 20$

$\therefore$  probability for  $\chi \leq 1$   
 ${}^{20}C_0 (0.02)^0 (0.98)^{20} + {}^{20}C_1 (0.02)^1 (0.98)^{19}$   
 $= 0.6676 + 0.2725$   
 $= 0.9401$   
 $1 - 0.9401 = 0.0599$

$\therefore P = 0.9401 \times 0.0599 \times 2$   
 $= 0.1126$

BP

Q5 (a) 2. Under **Scheme 1**, *Grow Well* tries to achieve a target acceptance rate of at least 96% of all truckloads. To the nearest 0.5%, find the greatest percentage defective rate for punnets that would enable *Grow Well* to achieve its target acceptance rate.

$$\begin{aligned} & {}^{20}C_0 (\chi)^0 (1 - \chi)^{20} + {}^{20}C_1 (\chi)^1 (1 - \chi)^{19} \\ & = 0.96 \\ & (1 - \chi)^{20} 20 \times (1 - \chi)^{19} = 0.96 \\ & (1 - \chi)^{19} (1 + 19\chi) = 0.96 \\ & \text{use bisection method with } \chi_1 = 0.01 \quad \chi_0 = 0.02 \end{aligned}$$

$$\begin{aligned} & \text{When } \chi = 0.02 \\ & f(\chi) = (1 - \chi)^{19} (1 + 19\chi) - 0.96 \\ & = -0.0199 < 0 \end{aligned}$$

$$\begin{aligned} & \text{When } \chi = 0.01 \\ & f(\chi) = 0.023 > 0 \end{aligned}$$

$$\begin{aligned} & \text{When } \chi = 0.015 \\ & f(\chi) = 4.254 \times 10^{-3} > 0 \\ & \therefore 0.015 < \chi < 0.02 \end{aligned}$$

$$\begin{aligned} & \text{When } \chi = 0.0175 \\ & f(\chi) = -7.231 \times 10^{-3} < 0 \\ & \therefore 0.015 < \chi < 0.0175 \end{aligned}$$

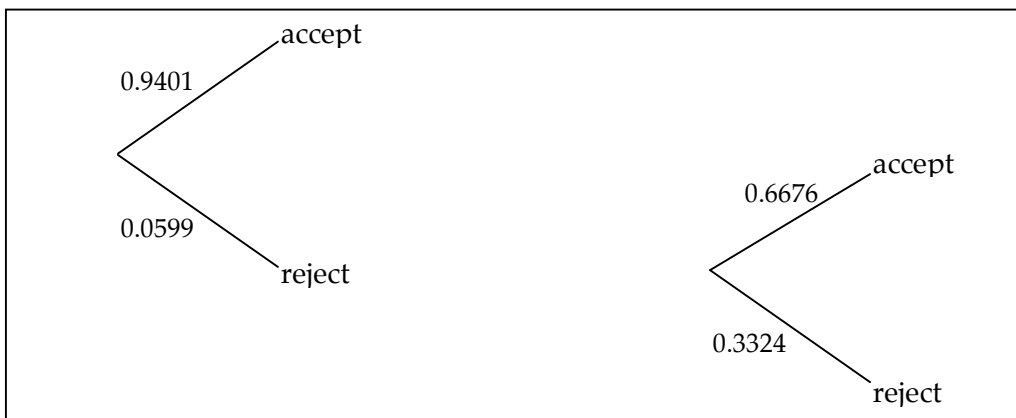
$$\begin{aligned} & \text{When } \chi = 0.01625 \\ & f(\chi) = -1.335 \times 10^{-3} < 0 \\ & \therefore 0.015 < \chi < 0.01625 \end{aligned}$$

$$\begin{aligned} & \text{When } \chi = 0.015625 \\ & f(\chi) = 1.499 \times 10^{-3} > 0 \\ & \therefore 0.015625 < \chi < 0.01625 \end{aligned}$$

$$\begin{aligned} & \therefore \text{to the nearest } 0.5\% \\ & \chi \approx 1.5\% \end{aligned}$$

BP

Q5 (a) 3. Under **Scheme 2**, calculate the proportion of accepted truckloads that would be accepted on the second sample.



Continued over.

$$P(\text{accept}) = 0.9401 + 0.0599 \times 0.6676$$

$$= 0.9801$$

$$P(\text{accept second}) / P(\text{accept})$$

$$= 0.0599 \times 0.6676 \div 0.9801$$

$$= 0.0408$$

4.08% of accepted trucks were accepted on the second sample.

AE

- (b) Find the largest value of  $n$  that would achieve an acceptance rate of at least 91% (to the nearest 1%) under **Scheme 3**, of all truckloads from *Quality Plants*.

$$TL = \frac{1}{150} = 6.667 \times 10^{-3}$$

$$(1-\pi)^n + n(\pi)(1-\pi)^{n-1}$$

$$0.9933^n + 0.0067n \times 0.9933^{n-1}$$

$$= 0.91$$

$$f(n) = (1-\pi)^n + n(\pi)(1-\pi)^{n-1} - 0.91$$

$$f'(n) = \ln(1-\pi)(1-\pi)^n + \pi(1-\pi)^{n-1} + \ln(1-\pi)(n\pi)(1-\pi)^{n-1}$$

$$\chi_0 = 40$$

$$\chi_1 = 84.13$$

$$\chi_2 = 75.02$$

$$\chi_3 = 74.78$$

$$\chi_4 = 74.78$$

$$\chi_5 = 74.78$$

$$\chi = 74.78 \approx 75$$

Largest value of  $n = 75$

Use Newton Raphson method  
Calculate:

AE

## QUESTION SIX

- (a) (1) What features of the time series should be considered in setting up this bonus payment scheme for managers?

The time series has periodic variation eg. A certain day of the week (as shown in table 2, Saturday) always had the higher value than the moving mean, the seasonal variation should be taken into account and the expected value is adjusted for each day of the week.

NS

- (2) Explain how the daily sales targets could be calculated.

Using the mean value of sales on that day (eg. Saturday) of many previous weeks as target.

NS

The information provided by the candidate is correct, but is insufficient to meet the requirements. An implied comment about trend was needed plus mention of all of the following at least once: seasonal, forecast, adjust for day, inflate by 5%.

## QUESTION SIX

- (b) (1) What effect does the high sales on Labour Day 2003 have on the (centred) moving mean for the daily retail sales?

It increases the moving mean of 6 adjacent days intensively.

- (2) How would you allow for the high sales figures for Labour Days in the calculation of a sales forecast for Mondays?

I would calculate the sales forecast without using this value on the Labour Day.

BS (both points 1 and 2 required)

- (c) Using the given information, forecast the sales for Tuesday 7<sup>th</sup> December 2004. You must make clear the method you are using to make your forecast and justify your reasoning.

$$y = 0.91x + 225.21$$

at Dec 2004,  $x = 42$

$$y = 0.91 \times 42 + 225.21 = 263.43$$

Seasonal variation for December

$$[(357.3 - 230.8) + (376.3 - 242.9)] \div 2 = 129.95$$

Seasonal variation for Tuesday

$$[(5.4 - 6.7) + (6.1 - 6.2) + (5.7 - 9.2) + (6.7 - 9.4) + (6.2 - 11.7)] \div 5 = -2.62$$

$$\therefore \text{the forecast for Tuesday 7}^{\text{th}} \text{ Dec 2004 is } \frac{263.43 + 129.95}{30} - 2.62$$

$$= 10.49 \text{ thousands of dollars}$$

BM

- (d) Describe two limitations of the forecast you made in part (c).

This forecast uses the function  $y = 0.91 + 215.21$  which may not still hold at time of Dec, 2004.

This forecast is incapable of predicting any random variation created by unpredictable events eg. Any political, cultural or natural events.

NS

Comments on the validity of the forecast were not sufficient to meet the standard for the award of AC. The candidate needed to comment on at least two distinct points, for example:

- That seasonal effect calculations were based on a limited amount of data.
- That forecasts were some time in advance of the data given.

**Sample of assessed candidate work - Performance Descriptor 3: Performance Category 9**

Note: Although this candidate achieved in seven areas, only six of these meet the criteria for a Performance Descriptor 2. The AT awarded meets the criteria for BM, but the AC awarded does not match any of the criteria for Performance Descriptor 2. Refer to the assessment schedule for marking codes.

**QUESTION ONE**

(e) Find a 95% confidence interval for the percentage support of all customers for this proposal.

$$z = 1.96$$

$$\frac{46}{200} - 1.96 \sqrt{\frac{\frac{46}{200} \left(1 - \frac{46}{200}\right)}{200}} < \pi < \frac{46}{200} + 1.96 \sqrt{\frac{\frac{46}{200} \left(1 - \frac{46}{200}\right)}{200}}$$

$$\frac{46}{200} - 0.0583 < \pi < \frac{46}{200} + 0.0583$$

$$0.1717 < \pi < 0.2883$$

BP

(f) Assuming that the percentage support of all customers is the value given by the lower limit of the interval in part (a), calculate the probability that the level of support from another sample of 200 customers is at least 25%.

$$\pi = 0.1717 \quad 0.25 \times 200 = 50$$

$$P(N \geq 50) = \sum_{50}^{200} B(x, 200, 0.1717)$$

$$n\pi = 34.34 \geq 5 \quad n(1\pi) = 165.55 \geq 5$$

$\therefore$  normal approx to Binomial

$$\mu = 34.34$$

$$\sigma = \sqrt{200 \times 0.1717 \times (1 - 0.1717)} = 5.33$$

$$Z = \frac{49.5 - 34.34}{5.33}$$

$$= 2.844$$

$$\begin{aligned} P(N \geq 50) &= P(Z > 2.844) \\ &= 0.5 - 0.4977 \\ &= 0.0023 \end{aligned}$$

[NB. Candidate's graph omitted.]

BP

- (g) NAILS claims that at least 30% of all customers support this proposal. Can this claim be justified? Give a reason for your answer.

No it is not justified.  
30% is not in the 95% C.I. for the proportion that support the proposed service. There is no evidence to show that 30% support it.

BS

- (h) The manager of NAILS requests that a second survey be conducted so that the width of the interval in (a) is reduced by 40%. How many customers should be interviewed in the second survey?

$$0.02332 = 0.4 \times 0.0583$$

$$0.02332 \geq 1.96 \sqrt{\frac{46}{200} \left(1 - \frac{46}{200}\right)}$$

$$\left(\frac{0.02332}{1.96}\right)^2 \geq \frac{46}{200} \left(1 - \frac{46}{200}\right)$$

$$n \geq \frac{46 \left(1 - \frac{46}{200}\right)}{\left(\frac{0.02332}{1.96}\right)^2}$$

$$\geq 1251.05$$

$$n = 1252$$

N – Marker has circled errors.

## QUESTION TWO

- (i) Find the third equation  
(ii) Find the general solution for this system of equations in terms of  $c$ .

$$\begin{aligned} 270a + 324b + 315c &= 9315 \\ 30a + 36b + 35c &= 1035 \quad (1) \\ 30a + 24b + 25c &= 477 \quad (2) \\ 30a + 30b + 30c &= 756 \quad (3) \end{aligned}$$

$$c = \frac{756 - 30a - 30b}{30}$$

$$= \frac{756}{30} - a - b$$

$$c = 25.2 - a - b$$

$$(1) - (2) \quad 12b + 10c = 558 \quad (4)$$

$$(2) - (3) + 6b + 5c = +279 \quad (5)$$

$$2 \times (5) \quad 12b + 10c = 558 \quad (5)^1$$

$$(4) - (5) \quad 0 + 0 = 0$$

BM – third equation (1) correctly deduced.

N – Equation 4 should have been rearranged to give  $b$  in terms of  $c$  then find  $a$  in terms of  $c$ .

(iii) Comment on the practical consequences of the solution in part (ii).

Because  $0=0$  is always true it means the planes (from above) have a common line of intersection

[NB. Candidate's graph omitted.]

The practical consequences are that the NAILS store is able to make any amount of each compost as long it is on this line.

If a large amount of all-purpose compost was needed, less of the other two would be made and all the Nitrogen, Phosphorus and potassium could be used if the amount of all-purpose was on this line.

AC

### QUESTION THREE

Write a report (approximately one page long) to the general manager of NAILS that summarises statistical output given below. Include some sales predictions in your report.

In the two years, from January 2001 to December 2002, the effect on your promotional expenditure has been investigated. Your expenditure has been compared with your relative sales and it has been found that there is a positive relationship between the amount you invest, and the amount of revenue you receive. We have found that your expenditure has been most profitable when you spend between \$15000 and \$30000. When you are spending this amount you are receiving over 600% back, on average, as revenue. It is because of this that I recommend you spend an average of \$30000 on promotional resources. You will receive on average \$163300. Any more than this and your sales revenue will decrease in proportion to your expenditure.

NS

The marker code indicates that the candidate's explanation is not wrong, but there is insufficient evidence to meet the requirements for awarding a grade.

## QUESTION FOUR

- (e) The management of NAILS wants to know how many bottles of both POW and ZAP should be produced. The preliminary estimates of their potential profitability are \$20 per bottle of POW and \$10 per bottle of ZAP. Perform an appropriate analysis and make a recommendation to management about the amount of POW and ZAP that could be produced to maximize profit.

$$30P + 20Z \leq 1500$$

$$P \leq 35$$

$$P + Z \leq 65$$

$$P \geq 15$$

$$Z \geq 15$$

$$\$P_x = 20P + 10Z$$

$$Z = \frac{P_x - 20P}{10}$$

$$= -2P + C \quad m = \frac{-2}{1}$$

max profit when  $P = 35$

$$30(35) + 20Z = 1500$$

$$Z = \frac{1500 - 30(35)}{20}$$

$$= 22.5 \quad = 22$$

$$\begin{aligned} \$P_x &= 20(35) + 10(22.5) \\ &= \$920 \end{aligned}$$

I recommend producing 35 bottles of POW and 22 of ZAP this will provide maximum profit.

*[Candidate's graph omitted]*

NS – Only one of the two possible solutions has been stated.

Q4 (b) Suppose that the profit of \$20 per bottle of POW was an overestimate and it was in reality on \$15 per bottle. How does that change your production recommendation in part (a)?

$\therefore P_x = 15P + 10Z$   

$$Z = \frac{P_x - 20P}{10}$$

$$= \frac{-3}{2}P + C \quad m = \frac{-3}{2}$$
 $\therefore \text{max profit along line } 30P + 20Z = 1500 \text{ between values of } P = 20 \text{ and } P = 35$ 

MEI

 I would recommend producing between 20 and 35 inclusive of POW, as long as ZAP is produced such that  $Z = 75 - \frac{3}{2}P$ .  
 I.e. 32 bottles of POW, 27 bottles of ZAP.

AT

The candidate has made a minor error, which was ignored. Sufficient evidence has been provided to meet the requirements for AT.

(c) Suppose  $x$  is the number of bottles of POW that should be produced to meet management's request and maximize the profit. Find an implicit equation for  $x$ .

$\chi : 9 \ln(\chi)$ 

$$\chi = 9 \ln \chi$$

$$0 = 9 \ln \chi - \chi$$

$$0 = e^9 \chi + e^{-\chi}$$

$$= 8103 \chi + e^{-\chi}$$

$$\frac{dy}{dx} = 8103 - e^{-\chi}$$

$$\chi = \chi_0 - \frac{8103\chi_0 + e^{-\chi_0}}{8103 - e^{-\chi_0}} e^{-\chi_0}$$

Marker has circled errors.

(d) Solve the equation derived in part (c) and make a recommendation to management about the amount of POW and ZAP that could be produced to maximize profit.

$\chi_0 = 30$ 

$$\chi = 30 - \frac{8103 \times 30 + e^{-30}}{8103 - e^{-30}}$$

$$= 30.000$$
 $\therefore \chi = 30$ 

$$9 \ln 30 = 30.6$$
 $\therefore \chi = 30, y = 30$

RAWW

The marker code indicates that the candidate has arrived at the correct answer but the working is wrong.

QUESTION FIVE

(d) Two percent of punnets from one supplier, *Grow Well*, are know to be defective.

- Two truckloads of punnets from *Grow Well* arrive at NAILS. Calculate the probability that exactly one truckload of the two is accepted.

$$\begin{aligned}
 n &= 20 & n\pi &= 0.4 \\
 \pi &= 0.02 & \therefore & \text{poisson approx to Binomial} \\
 P(\text{truck 1 not accepted}) &= P(N \geq 2) \\
 &= \sum_2^{20} B(x, 20, 0.02) \\
 \lambda &= n\pi = 0.4
 \end{aligned}$$

$$\begin{aligned}
 P(N \geq 2) &= 1 - P(N \leq 1) \\
 &= 1 - \sum_0^1 P(x, 0.6) \\
 &\quad 0.5488 + 0.3293 \\
 &= 1 - (0.6703 + 0.2681) \\
 &= 0.0616 \quad (0.1219)
 \end{aligned}$$

$$\begin{aligned}
 P(\text{NA, A or A, NA}) &= \frac{0.1070}{0.5781} \times 2 \\
 &= 0.1156 \\
 &\quad (0.2140)
 \end{aligned}$$

Marker has circled errors.

N – Not correct: 0.05781 should be doubled.

Calculations are incorrect. The candidate has not met the requirements.

2. Under **Scheme 1**, *Grow Well* tries to achieve a target acceptance rate of at least 96% of all truckloads. To the nearest 0.5%, find the greatest percentage defective rate for punnets that would enable *Grow Well* to achieve its target acceptance rate.

To accept 96% 0.04 need to be not accepted  
0.04 need to have  $N \geq 2$ .

$\lambda = ?$

$P(\text{not accept}) = 0.04$

$$0.04 = 1 - \sum_0^1 P(x, \lambda)$$

$$\sum_0^1 P(x, \lambda) = 1 - 0.04$$

$$= 0.96$$

$$\begin{aligned} 0.96 &= \frac{\lambda^0 e^{-\lambda}}{0!} + \frac{\lambda^1 e^{-\lambda}}{1!} \\ &= e^{-\lambda} (\lambda^0 + \lambda^1) \\ &= e^{-\lambda} (1 + \lambda) \end{aligned}$$

$$0 = e^{-\lambda} + \lambda e^{-\lambda} - 0.96$$

$$0.96 = e^{-\lambda} + \lambda e^{-\lambda}$$

$$\ln 0.96 = -\lambda + \ln \lambda + -\lambda$$

$$-0.04082 = -2\lambda + \ln \lambda$$

$$0.96 = e^{-2\lambda} + \lambda$$

$$0 = e^{-2\lambda} + \lambda - 0.96$$

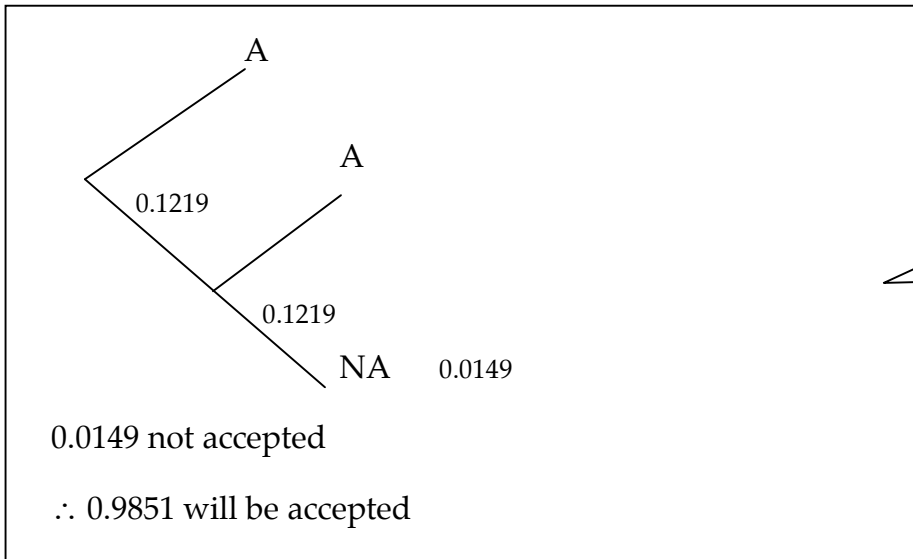
$$\therefore \lambda = 0.73$$

$\therefore$  to accept 96% the rate of defectiveness needs to be 2.7%

N – Not correct to take ln's of both sides.

Candidate has not been able to apply probability theory to solve a complex problem.

Q5 (a) 3. Under **Scheme 2**, calculate the proportion of accepted truckloads that would be accepted on the second sample.



Candidate failed to calculate a conditional probability.

(e) Find the largest value of  $n$  that would achieve an acceptance rate of at least 91% (to the nearest 1%) under **Scheme 3**, of all truckloads from *Quality Plants*.

$$\frac{2}{150} n > 1.696 \sqrt{\frac{1}{150} \left(1 - \frac{1}{150}\right)}$$

$$\left(\frac{2}{150}\right)^2 > \frac{1}{150} \left(1 - \frac{1}{150}\right)$$

$$n > \frac{\frac{1}{150} \left(1 - \frac{1}{150}\right)}{\left(\frac{2}{150}\right)^2}$$

$n > 107$   
 $n = 108$

N

First line of candidate's answer is incorrect.

QUESTION SIX

- (a) (1) What features of the time series should be considered in setting up this bonus payment scheme for managers?

The seasonally adjusted centered moving mean

NS

- (2) Explain how the daily sales targets could be calculated.

Calculate the seasonal adjustment by working out the average difference from the C.M.M for a large number of seasons for each season (day) calculate the amount that must be exceeded by multiplying the C.M.M plus the seasonal adjustment by 1.05. This is the daily sales target, to get the bonus.

NS

- (b) (1) What effect does the high sales on Labour Day 2003 have on the (centred) moving mean for the daily retail sales?

It is abnormally high for a Monday. The C.M.M before Labour day is steadily climbing and after Labour day it sharply decreases.

NS

- (2) How would you allow for the high sales figures for Labour Days in the calculation of a sales forecast for Mondays?

Use a large number of Mondays to calculate the seasonal adjustment for Mondays.

NS

The evidence presented is correct, but the candidate needed to provide a fuller explanation to meet the requirements.

- (c) Using the given information, forecast the sales for Tuesday 7<sup>th</sup> December 2004. You must make clear the method you are using to make your forecast and justify your reasoning.

Seasonal adjustment for Tuesdays:

$$= ((5.4 - 6.7) + (6.1 - 6.2) + (5.7 + 9.2) + (6.7 - 9.4) + (6.2 - 11.7))$$

$$\begin{aligned} \text{S.A (tue)} &= \frac{-13.1}{5} \\ &= -2.62 \end{aligned}$$

$$D - 01 = 6$$

$$D - 02 = 18$$

$$D - 03 = 30$$

$$D - 04 = 42$$

$$\begin{aligned} \therefore \text{ for Dec 2004 } S &= 0.91(42) + 225.21 \\ &= 263.43 \end{aligned}$$

$\therefore$  Ave sales per day in Dec 2004 ie. C.M.M

$$\frac{263.43}{31} = 8.4977$$

$$\begin{aligned} \therefore \text{ sales for 7}^{\text{th}} \text{ December 2004} \\ &= 8.4972 - 2.62 \\ &\approx 5.88 \end{aligned}$$

$\therefore$  in 7<sup>th</sup> Dec sales is \$5880

BM

- (d) Describe two limitations of the forecast you made in part (c).

December is the month of Christmas

$\therefore$  People will be shopping for presents. They may buy tools for fathers etc.

$\therefore$  the actual value may be much higher than calculated.

The seasonal adjustment was only calculated from 5 weeks. For a more accurate seasonal adjustment a larger sample is needed ie a whole year.

NS