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93201Q



NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

Scholarship 2008 Statistics and Modelling

2.00 pm Saturday 22 November 2008
Time allowed: Three hours
Total marks: 48

QUESTION BOOKLET

Pull out the Formulae and Tables Booklet S–STATF from the centre of this booklet.

There are SIX questions in this booklet. Answer ALL questions.

Write your answers in the Answer Booklet 93201A.

Show ALL working. Start your answer to each question on a new page. Carefully number each question and part question.

There is a grid provided on page 25 of the Answer Booklet for answering Question Three (b)(i), and another grid provided on page 26 for answering Question Six (a).

Check that this booklet has pages 2–10 in the correct order and that none of these pages is blank.

YOU MAY KEEP THIS BOOKLET AT THE END OF THE EXAMINATION.

You have three hours to complete this examination.

All questions are based on activities associated with a hotel owned and operated by a company called Statsmod Enterprises.

QUESTION ONE: HOTEL RESTAURANT SPENDING ANALYSIS (8 marks)

- (a) In an analysis of the spending at the restaurant, Statsmod Enterprises wishes to estimate the mean amount spent per person to within \$4, with 99% confidence.

Calculate the required minimum sample size n if the standard deviation of the amount spent per person has been estimated to be \$9.53.

- (b) The restaurant has a maximum capacity of 80 diners. Previous analysis has shown that the mean total nightly takings when the restaurant is full is \$4800. A random sample of the size n , calculated in part (a), gave a mean amount spent per person of \$56.24.

Does this sample provide evidence, at the 95% level of confidence, to suggest that the mean amount spent per person has decreased? Justify your answer.

- (c) Statsmod Enterprises wishes to compare the mean amount spent per diner between lunch and dinner (using an identical menu) to see if it can be claimed that these means are different.

Describe how to carry out a statistical procedure that would allow Statsmod to make a conclusion. State any assumption(s) that would be needed.

QUESTION TWO: GUEST SATISFACTION SURVEY (8 marks)

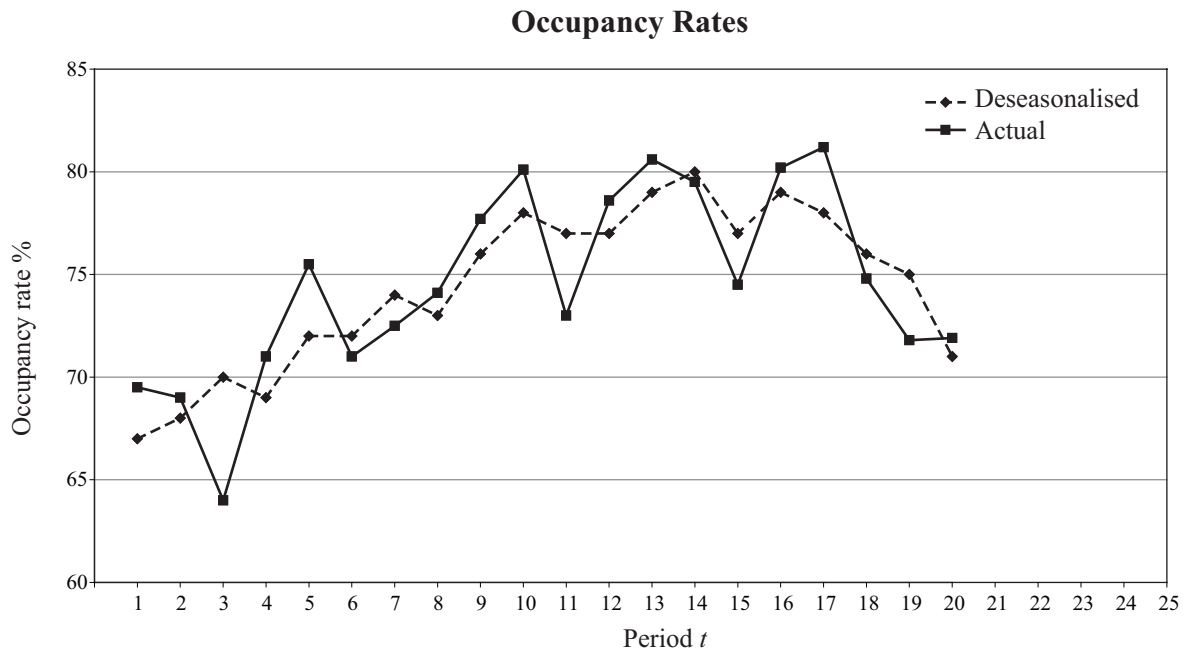
- (a) In order to carry out a guest satisfaction survey, a systematic sample was taken from the guest database. Explain how this sample could be taken, and give both an advantage and a disadvantage of using this method.
- (b) Suppose a pilot survey of 50 guests gave a 78% satisfaction rate with the room service. Construct a 95% confidence interval and use it to calculate two possible minimum sample sizes that would achieve an estimate of the true satisfaction rate to within 3%, with 95% confidence. Explain the difference between your answers.
- (c) On average, from similar previous satisfaction surveys, 5% of the questionnaires are completed incorrectly. What is the maximum number of incorrectly completed questionnaires, out of a random sample of 56 returned questionnaires, that would be expected 95% of the time?

QUESTION THREE: OCCUPANCY RATE INVESTIGATION (8 marks)

Statsmod Enterprises wishes to investigate the trend in the occupancy rate of its hotel in the period 2003 to 2007. Data for the actual occupancy rate for each quarter of the years 2003 to 2007 were collected as shown in the following table. Period $t = 1$ is the first quarter in 2003, period $t = 2$ is the second quarter of 2003, and so on. Deseasonalised occupancy rates were calculated to the nearest whole number and these are also shown in the table.

Period t	Actual occupancy rate (%)	Deseasonalised occupancy rate (%)
1	69.5	67
2	69.0	68
3	64.0	70
4	71.0	69
5	75.5	72
6	71.0	72
7	72.5	74
8	74.1	73
9	77.7	76
10	80.1	78
11	73.0	77
12	78.6	77
13	80.6	79
14	79.5	80
15	74.5	77
16	80.2	79
17	81.2	78
18	74.8	76
19	71.8	75
20	71.9	71

From the data in the table the following graph was obtained.



A trend line was fitted to the deseasonalised data. Its equation is $y = 0.4496 t + 69.679$, where y is the deseasonalised occupancy rate as a percentage.

- (a) Write a short paragraph to describe occupancy rates over the years 2003 to 2007.
- (b) Obtain a forecast for the occupancy rate in the first quarter of 2008 by each of the following methods:
- Extrapolate the deseasonalised occupancy rates (%) corresponding to $t = 16, 17, 18, 19$ and 20 . Use the grid provided on page 25 of the Answer Booklet.
 - Use the fitted trend line.
- (c) Discuss any differences between your answers in part (b).

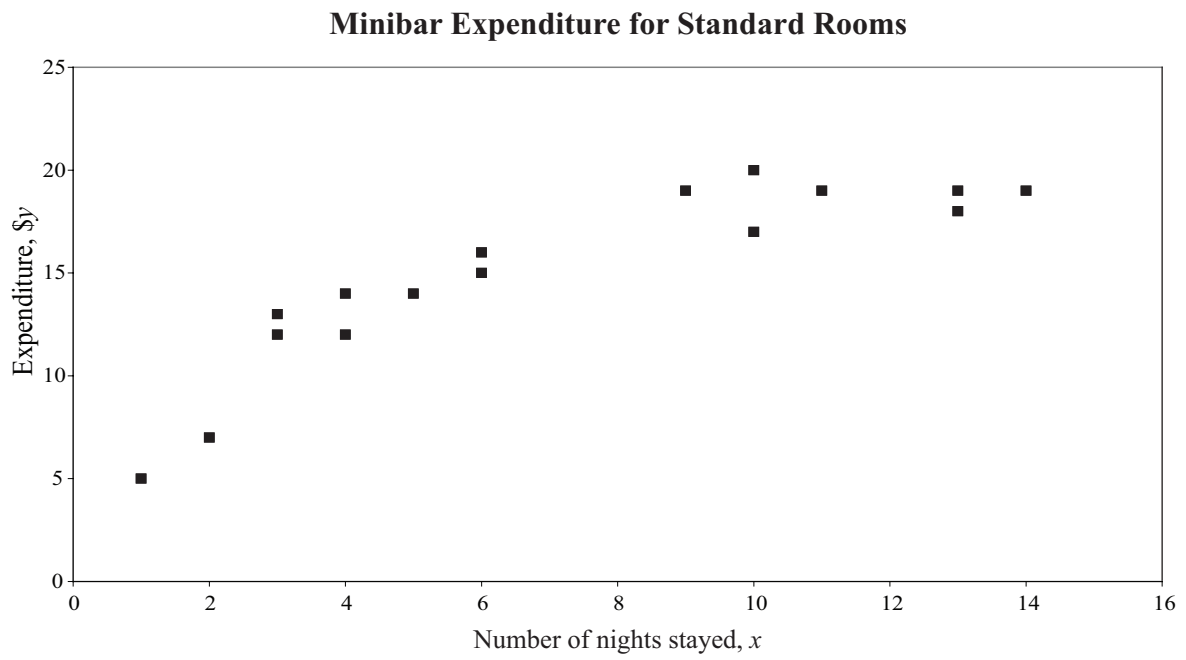
QUESTION FOUR: MINIBAR EXPENDITURE ANALYSIS (8 marks)

Statsmod's hotel has a minibar available in each of its standard and deluxe rooms. A minibar is a small selection of drinks and snack food for sale in the hotel room. Data about expenditure on the minibar were collected from a random sample of occupants in 16 standard rooms and a random sample of occupants in 16 deluxe rooms, and statistics were obtained from the data, as shown in the following tables.

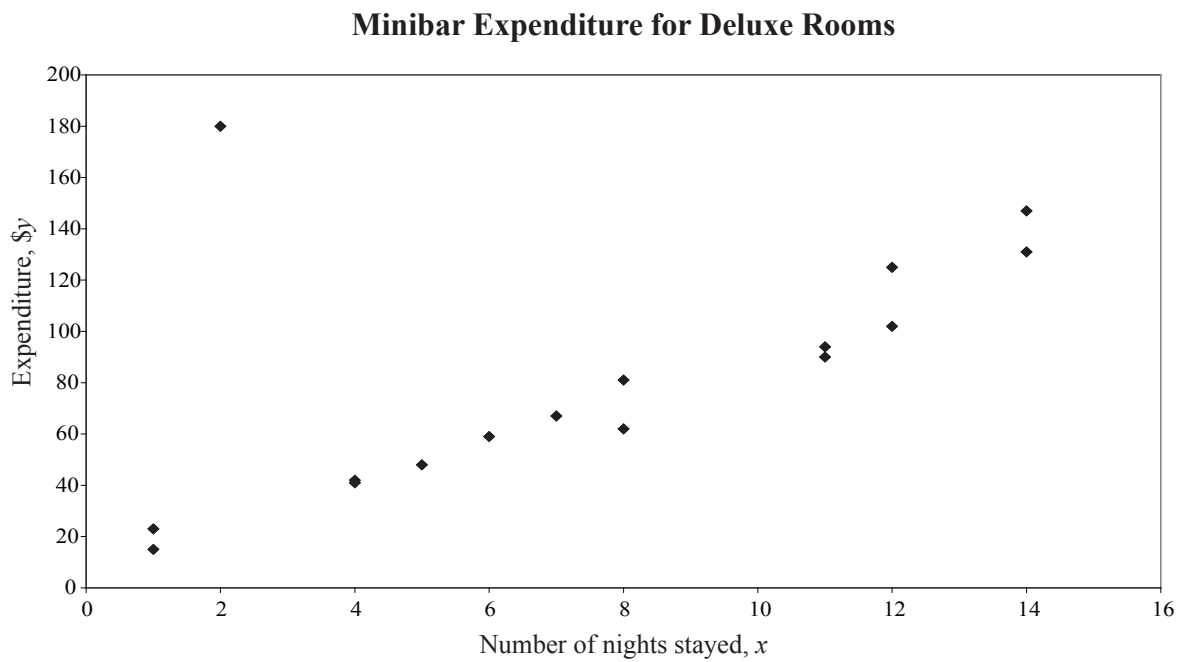
Number of nights stayed, x	Minibar expenditure for standard room, \$ y
1	5
2	7
3	12
3	13
4	12
4	14
5	14
6	15
6	16
9	19
10	17
10	20
11	19
13	19
13	18
14	19
Mean	14.94
Standard Deviation	4.40

Number of nights stayed, x	Minibar expenditure for deluxe room, \$ y
1	15
1	23
2	180
4	42
4	41
5	48
6	59
7	67
8	62
8	81
11	90
11	94
12	102
12	125
14	131
14	147
Mean	81.69
Standard Deviation	46.37

A scatter plot of the data for standard rooms is shown below.



A scatter plot of the data for deluxe rooms is shown below.



Two regression lines, one of which uses a log (base e) transformation of x , were fitted to the standard rooms data. The equations of the regression lines and their associated R^2 values are:

$$y = 5.58 \ln x + 5.23 \text{ with } R^2 = 0.9274$$

$$y = 0.90 x + 8.53 \text{ with } R^2 = 0.7674$$

Two regression lines were fitted to the deluxe rooms data; one of them to the complete set of 16 observations, and the other to 15 observations. The equations of the regression lines (not necessarily respectively) and their associated R^2 values are:

$$y = 5.95 x + 37.03 \text{ with } R^2 = 0.3277$$

$$y = 8.85 x + 5.53 \text{ with } R^2 = 0.9483$$

Write a two-page essay that comments on the minibar expenditure by the occupants in the two types of room.

As part of your essay, by making an appropriate choice of regression line from those listed above, include a prediction for the amount spent on the minibar by occupants for a stay of seven nights for each type of room. Justify your choice of regression line. Comment on the validity of your predictions. State at least two other factors that could have an influence on the amount spent on the minibar and which could form part of a more extensive analysis.

QUESTION FIVE: BOOKINGS ANALYSIS (8 marks)

Accommodation enquiries may be made at Statsmod's hotel either by phone, through the internet, or by fax. Enquiries are received randomly with resulting bookings occurring randomly as well. An analysis of enquiries and bookings was carried out. Some conclusions from this analysis are summarised in the following table.

Type of enquiry	Probability of receiving this type of enquiry	Probability that a booking results from this type of enquiry
Phone	0.6	a
Internet	0.3	a²
Fax	0.1	a³

where **a** is a constant such that $0 < \mathbf{a} < 1$.

- (a) Calculate the probability that at least 25 enquiries out of 100 came through the internet.
- (b) Obtain an expression, in terms of **a**, that gives the probability that the 10th phone call resulted in the 5th booking from phone enquiries on that day. Assume that the phone calls occur independently.
- (c) By taking $\mathbf{a} = 0.7$, calculate the percentage of enquiries that result in no booking.
- (d) Suppose that, when a booking resulted, there was a 75% chance it was by phone.

Find the value of **a**.

QUESTION SIX: DESIGNING A NEW HOTEL (8 marks)

A new hotel for Statsmod Enterprises is to have only deluxe and standard rooms. In designing the new hotel, the following constraints need to apply:

- The maximum number of rooms is 65.
- The building cost budget for the rooms has a maximum of \$3 million. Building costs are \$45 000 for a standard room and \$60 000 for a deluxe room.
- When the number of standard rooms (x) is such that $10m < x \leq 10(m + 1)$, then the number of deluxe rooms (y) is such that $y \leq 5(m + 1)$, where $m = 0, 1, 2, 3, 4$.

The profit for deluxe rooms is 40% greater than that for standard rooms.

- (a) Assuming that all rooms are occupied, find the number of each type of room that should be built to ensure maximum profit.
A grid is provided on page 26 of the Answer Booklet to help you answer this question.
- (b) Statsmod Enterprises wants to ensure maximum profit and build the maximum number of 65 rooms, but cannot do so within the \$3 million building cost budget.

What is the smallest required increase in this budget?

- (c) Suppose the mean occupancy rate for deluxe rooms is 60% and that all three constraints still apply.

While still ensuring maximum profit, calculate the occupancy rate for standard rooms, which will give more than one solution for each type of room that should be built. Clearly state all of these solutions.