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90644

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

Level 3 Statistics and Modelling, 2006

90644 Solve equations

Credits: Four
2.00 pm Tuesday 21 November 2006

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Make sure you have a copy of the Formulae and Tables Booklet L3–STATF.

You should answer ALL the questions in this booklet.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–15 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

For Assessor's use only	**Achievement Criteria**	
Achievement	**Achievement with Merit**	**Achievement with Excellence**
Solve equations. ☐	Solve problems involving equations. ☐	Analyse or interpret the outcome or the process used to solve equations or linear programming problems. ☐
Overall Level of Performance ☐		

You are advised to spend 65 minutes answering the questions in this booklet.

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QUESTION ONE

Solve the system of equations

$$\begin{aligned} 2a + 2b &= c + 14 \\ 2a + 2c &= -5 \\ 4a - 4b - 3c &= -9 \end{aligned}$$

QUESTION TWO

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The equation $x^5 + 2x^2 - 5 = 0$ has a real root between 1 and 2.

Complete two iterations of either the Newton-Raphson method or the bisection method, to find an approximation for this root.

Show each iterate.

Use $x_0 = 1.0$ as the starting value if you use the Newton-Raphson method.

Use $x_0 = 1.0$ and $x_1 = 1.5$ as the starting values if you use the bisection method.

QUESTION THREE

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Marni makes and sells two types of scented soaps.
Her "Vitamin E & Chamomile" soap takes 6 minutes to make, and uses 20 grams of a fat and sodium mix.
Her "Aloe & Lanolin" soap takes 5 minutes to make, and uses 30 grams of the fat and sodium mix.

Each day Marni has 1500 grams of the fat and sodium mix available, and can work for 330 minutes. She must produce daily at least 15 "Vitamin E & Chamomile" soaps and at least 12 "Aloe & Lanolin" soaps to satisfy existing client orders. However, she has enough customer interest in her products to be able to sell any extra she produces.

Let x be the number of "Vitamin E & Chamomile" soaps produced per day and y be the number of "Aloe & Lanolin" soaps produced per day.

A linear programming problem for this situation has the following constraints:

A: $20x + 30y \leq 1500$

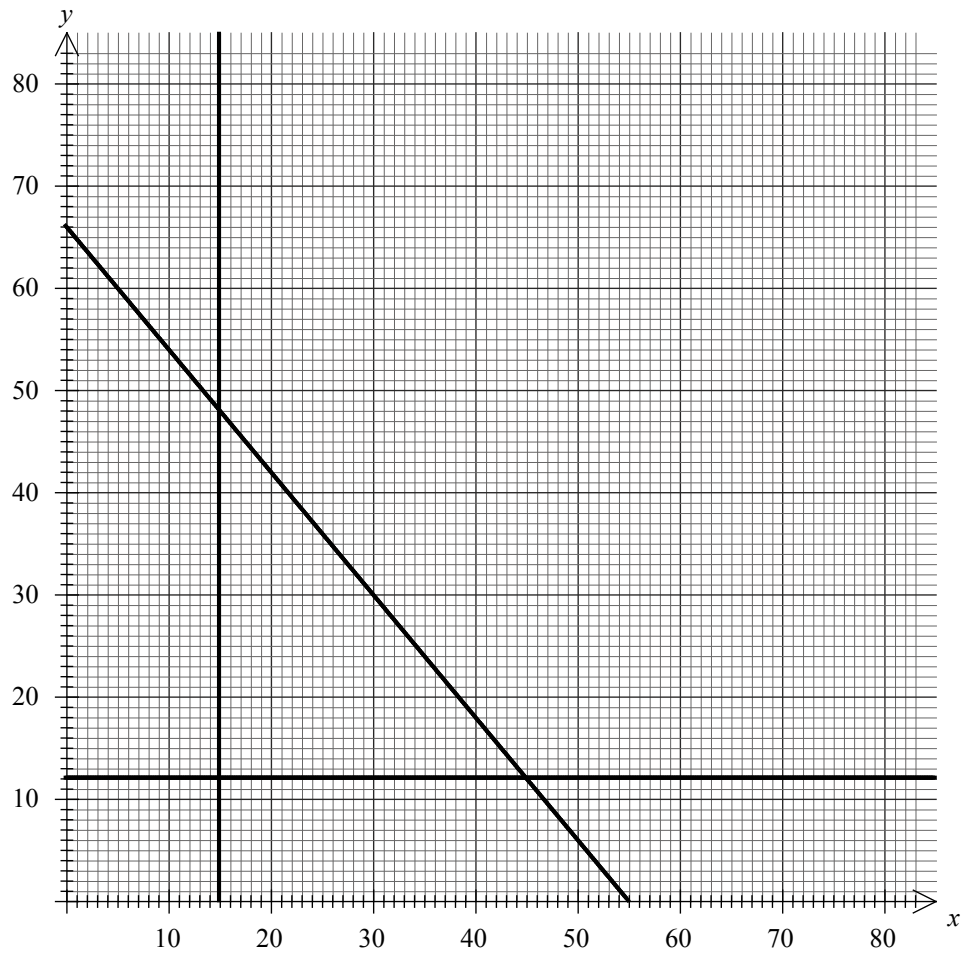
B: $6x + 5y \leq 330$

C: $x \geq 15$

D: $y \geq 12$

If you need to redraw this graph, use the grid on page 13.

(a) Draw these constraints on the axes below, and show the feasible region.
Three of the lines that you need have been drawn for you.



(b) Marni sells the "Vitamin E & Chamomile" soap for \$1.25 each, and the "Aloe & Lanolin" soap for \$1.40 each. Her income I (\$) is given by the equation $I = 1.25x + 1.4y$

Calculate how many of each soap Marni should make each day in order to maximise her income.

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QUESTION FOUR

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Marni's friend Jane makes and sells three different types of hand lotion. Over the last three days, she has kept a tally of how many containers of each type she has made. This is summarised in the table below. On the first day, she spent a total of 145 minutes making hand lotions, the second day, 130 minutes, and on the third day she spent two hours making hand lotions.

NUMBER OF EACH TYPE OF HAND LOTION MADE

Type of Hand Lotion	Number of containers		
	Day One	Day Two	Day Three
Dewberry	8	2	5
Vitamin E enriched	3	2	0
Lavender	6	10	8

Set up and solve a system of equations to find the type of hand lotion for which the time taken to produce each container is the smallest.

QUESTION FIVE

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A factory uses two machines. After maintenance work, it takes time for the machines to get back to 100% operating efficiency.

The operating efficiency of the first machine is modelled by the function $y = 0.8x^{1.5}$, where y is the operating efficiency and x the time (in minutes, from 0 up to 25 minutes) after the machine is switched on.

The operating efficiency of the second machine is modelled by the function $y = 5x - 25$, where y is the operating efficiency and x the time (in minutes, from 5 up to 25 minutes) after the machine is switched on.

The factory manager requires an accurate estimate of the first time both machines are operating at the same efficiency. The machines are switched on at the same time.
This time can be obtained by finding the smaller root of the equation $0.8x^{1.5} - 5x + 25 = 0$.

Use either the bisection method, or the Newton-Raphson method, to solve the equation to find the time required by the manager.

If you want to draw a graph, use the grid on page 14.

You must state your starting value(s), show the results of each iteration, and give your answer correct to two decimal places.

(Note that if $f(x) = 0.8x^{1.5} - 5x + 25$, then the derived function $f'(x) = 1.2x^{0.5} - 5$.)

QUESTION SIX

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Marni's friend Vili has a small business, making sun-shelters and tents for small children. The table below summarises the time it takes to produce each item, and the amount of material it uses.

	Production time (minutes)	Amount of material used (m^2)
Sun-shelter	30	2
Tent	40	5

Let x be the number of sun-shelters produced each week, and y be the number of tents produced each week.

Each week, Vili has available 30 hours to work on making these products, and a total of 190 square metres of material. Currently Vili has a steady order to produce at least 10 sun-shelters and 15 tents each week for a stall at the local market. Any extras he produces can always be sold to a local store.

(a) For each sun-shelter he produces, Vili makes a profit of $8, while he makes a profit of $12 for each tent.

(i) Write down the four constraints you would need to use in order to find the number of each product Vili should produce each week to maximise his profit.

(ii) Calculate how many sun-shelters and how many tents Vili should produce per week, in order to maximise his profit. The graph on the following page is provided to help you.

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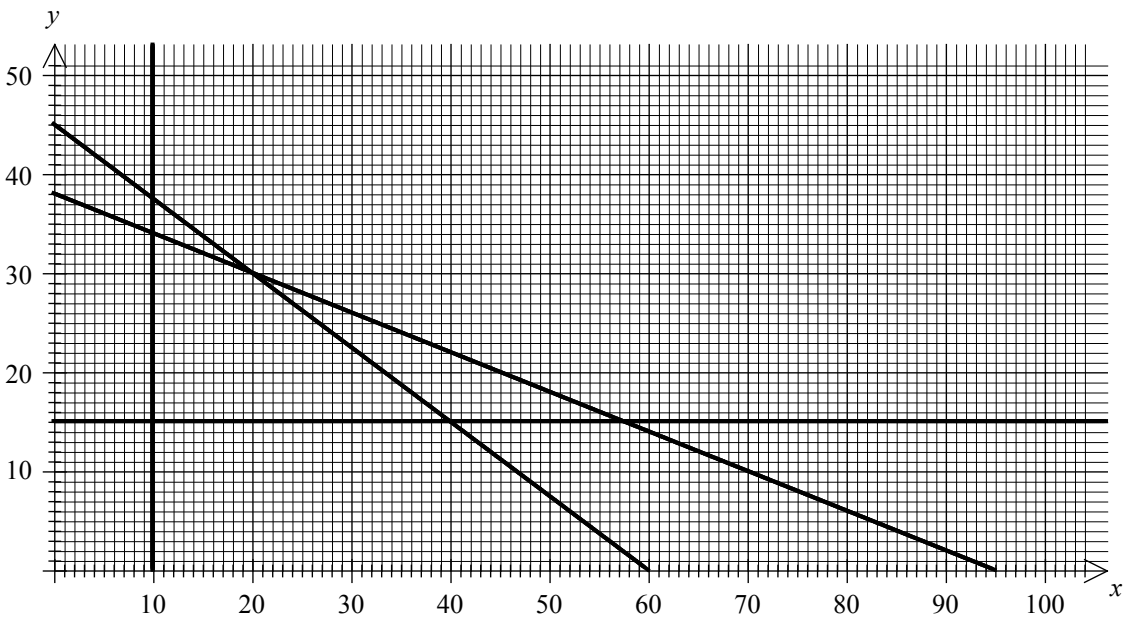

(b) Suppose Vili's profit for each sun-shelter becomes $9.

Explain how this changes the solution obtained in part (a)(ii). Give mathematical reasons to justify your answer.

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QUESTION SEVEN

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A student intends to use the bisection method to solve the equation $\frac{4}{3x-8} + 0.1x^2 = 0$.

He calculates the following set of values for the function $f(x) = \frac{4}{3x-8} + 0.1x^2$.

x	0	1	2	3
$f(x)$	−0.5	−0.7	−1.6	4.9

The student suggests that because $f(2)$ is negative and $f(3)$ is positive, a real root of $f(x) = 0$ must lie between $x = 2$ and $x = 3$.

Justify why, even though there is a change of sign in the function between $x = 2$ and $x = 3$, the equation $f(x) = 0$ does **not** have a real root in this interval.

QUESTION EIGHT

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Consider the following system of three equations in x, y and z.

$$2x + 4y + 5z = 17$$
$$4x + ay + 3z = b$$
$$8x + 7y + 13z = 40$$

Give values for a and b in the second equation that make this system consistent, but with an infinite set of solutions.

If you need to redraw the graph from Question Three, draw it on the grid below.

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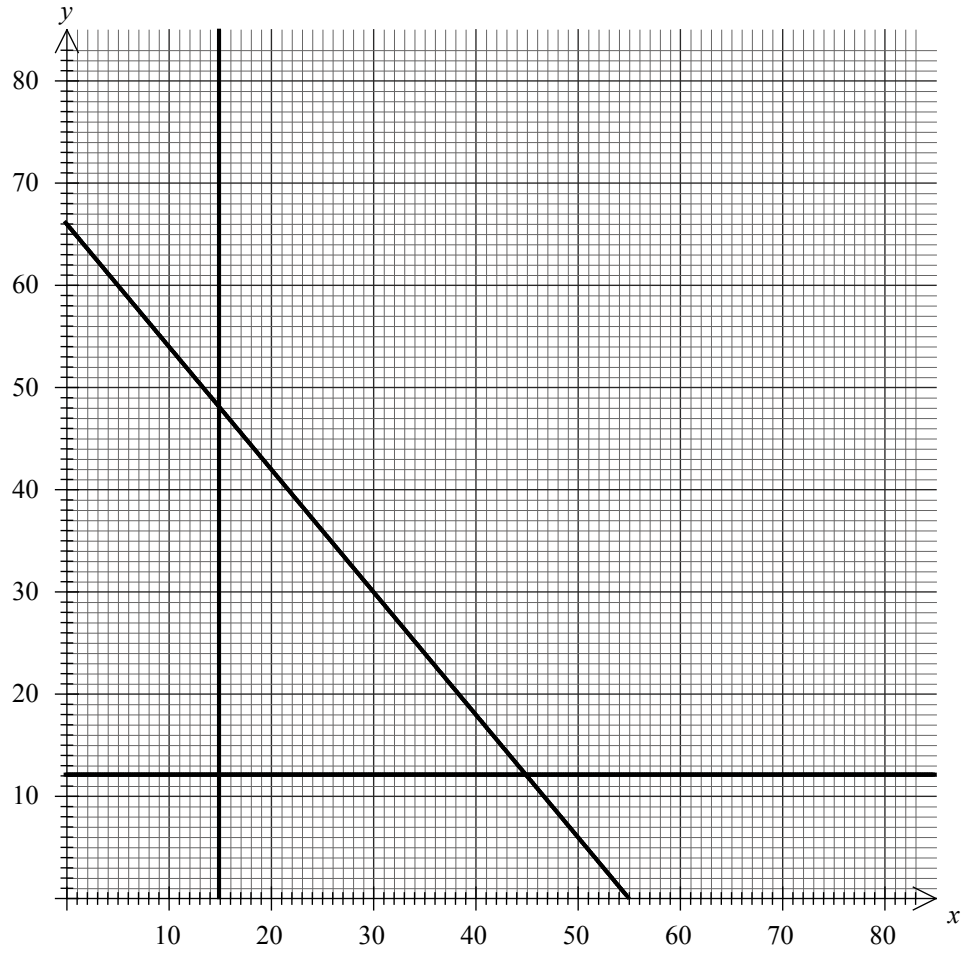

If you want to draw a graph as part of your answer to Question Five, use this grid.

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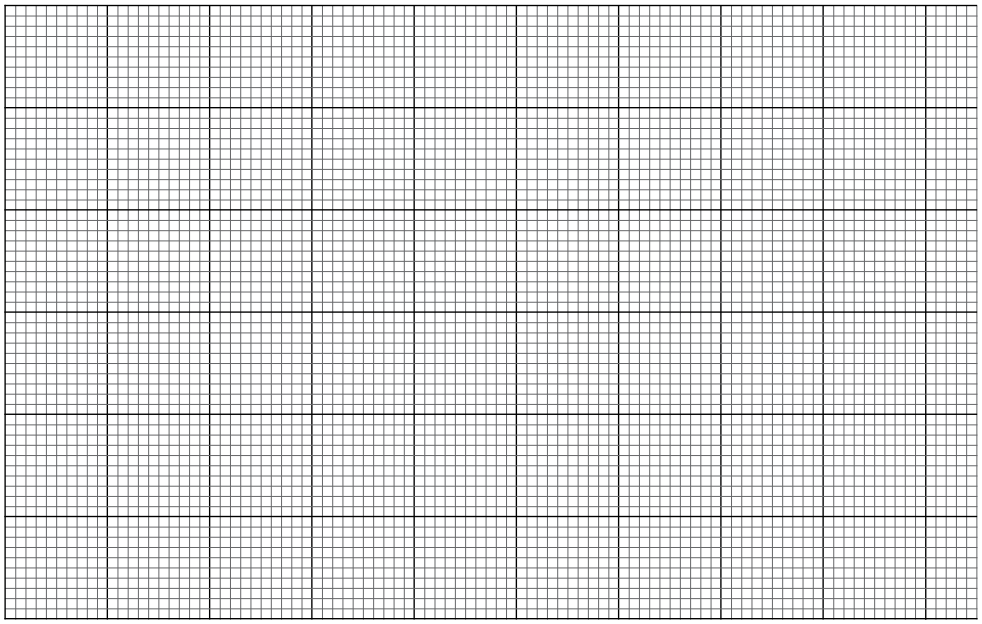

Extra paper for continuation of answers if required.
Clearly number the question.

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Question number	