



For Supervisor's use only

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90644



NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA



National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

Level 3 Statistics and Modelling, 2005

90644 Solve equations

Credits: Four

9.30 am Thursday 24 November 2005

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Make sure you have a copy of the Formulae and Tables Booklet L3-STATF.

You should answer ALL the questions in this booklet.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–20 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

<i>For Assessor's use only</i>		Achievement Criteria	
Achievement		Achievement with Merit	Achievement with Excellence
Solve equations.	<input type="checkbox"/>	Solve problems involving equations.	Analyse or interpret the outcome of the process used to solve the equations or linear programming problems. <input type="checkbox"/>
Overall Level of Performance		<input type="checkbox"/>	

QUESTION THREE

To supplement his income, Marty makes wooden frames for climbing roses, which he then sells to the local garden centres. Marty makes and sells two different-sized frames, a medium one and a large one.

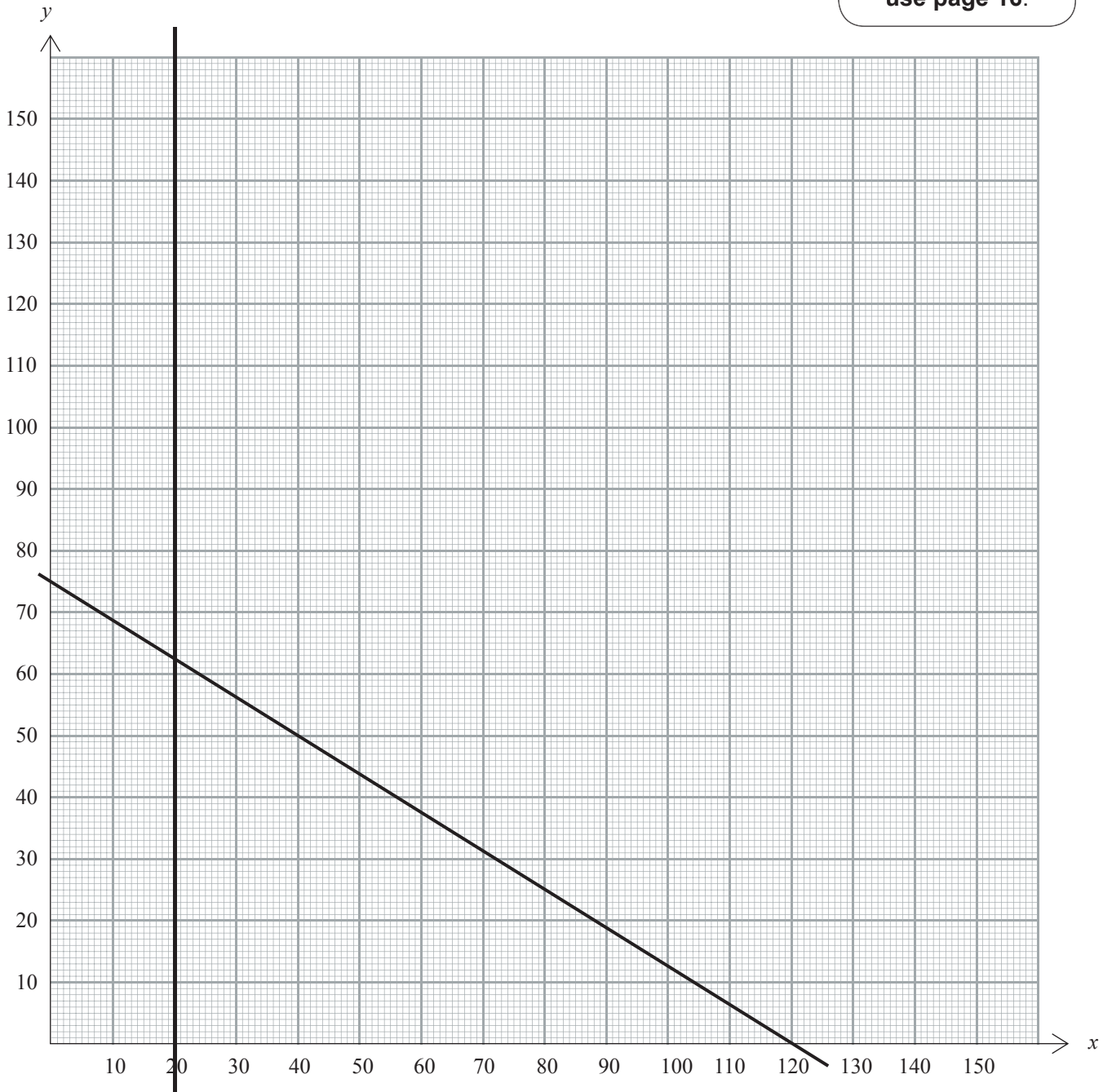
Let x represent the number of medium frames Marty makes and sells in a week, and y represent the number of large frames Marty makes and sells in a week.

The following inequations represent the weekly constraints in the production of Marty's frames:

$$7500x + 15000y \leq 1050000 \quad 5x + 8y \leq 600 \quad x \geq 20 \quad y \geq 40$$

- (a) Draw these constraints on the axes below, and show the feasible region.
Two of the lines that you need have been drawn for you.

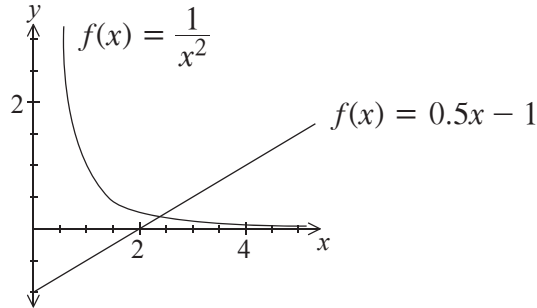
**If you need to
redraw this graph
use page 16.**



QUESTION FOUR

A student is completing an investigation of the bisection and the Newton-Raphson methods for solving non-linear equations.

- (a) The student decided to start by showing how these methods could be used to find the point of intersection of the graphs of the functions $f(x) = \frac{1}{x^2}$ and $f(x) = 0.5x - 1$.



In order to find this point, the solution of the equation $0.5x^3 - x^2 - 1 = 0$ has to be found.

Complete two iterations of either the bisection method or the Newton-Raphson method to find an approximation to the solution of this equation.

Show the results of each iteration.

Use $x_0 = 2.5$ as the starting value if you use the Newton-Raphson method.

Use $x_0 = 2$ and $x_1 = 3$ as the starting values if you use the bisection method.

QUESTION SEVEN

Maria decides she can produce rose frames that are similar to those Marty was making in Question Three, but they will be cheaper.

Maria calculates that each medium frame only needs a 6000 mm length of timber and can be built in four minutes, while a large frame only needs a 12000 mm length of timber and can be built in six minutes.

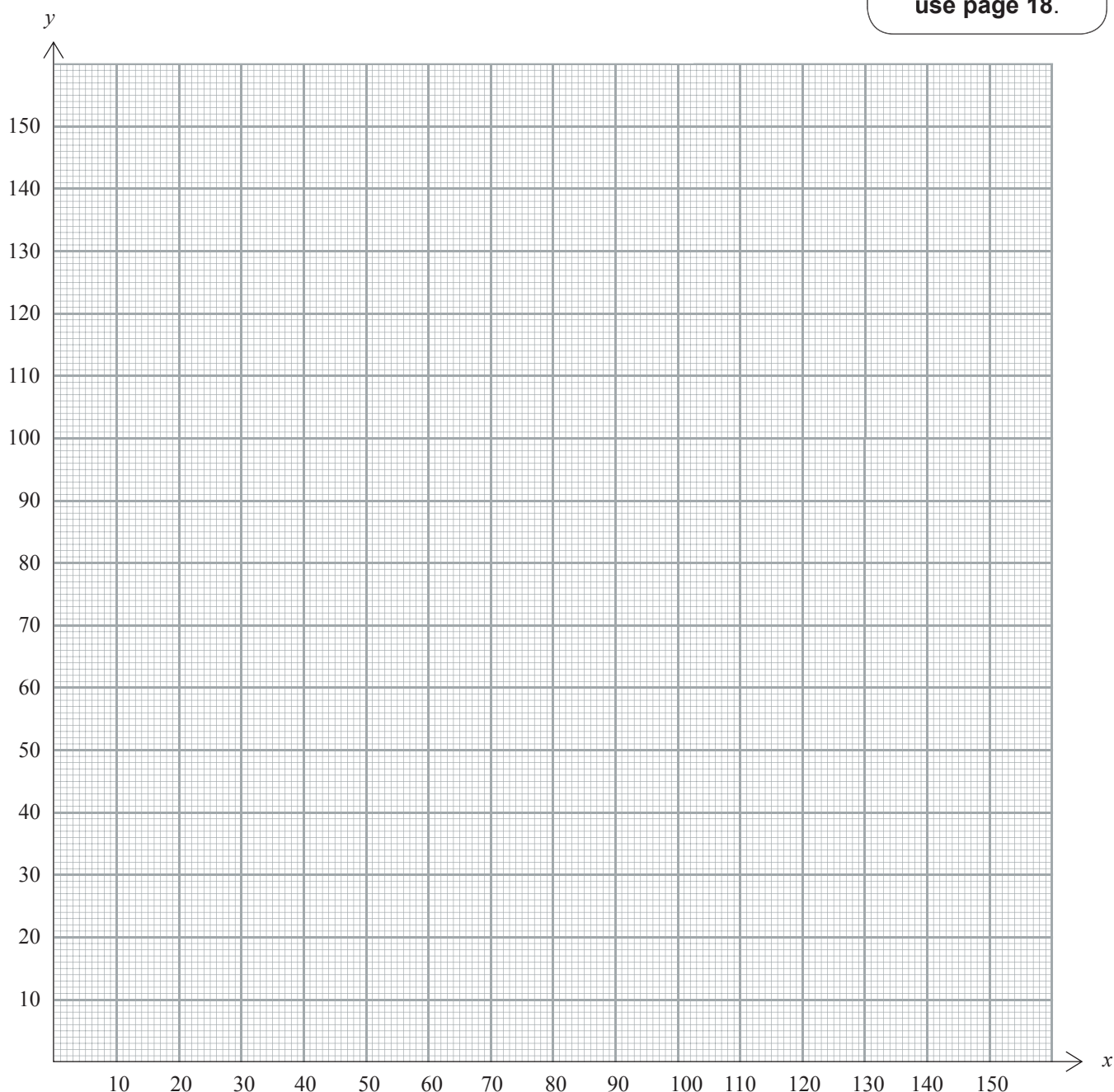
She has a regular supply of suitable timber, a total length of 900 metres per week. She also has a maximum of eight hours a week to build them.

Maria contacts the manager of a local chain of hardware stores. The manager is willing to stock Maria's frames if she can guarantee at least 20 of each size per week, but will take no more than 60 of each size per week.

The deal will give Maria a \$6 profit on each medium frame produced, and a \$10 profit on each large frame.

You may use the axes below to help you answer the following questions.

**If you need to
redraw this graph
use page 18.**



QUESTION NINE

The student in Question Four decides to compare the bisection method and the Newton-Raphson method by using both methods to find the positive fourth root of 26.

Suppose that this root is α .

To complete her investigation, the student uses both methods to solve the equation $x^4 - 26 = 0$.

To investigate the bisection method, the student begins by finding that α lies between $x = 2$ and $x = 3$.

Her first iteration establishes that α lies between $x = 2$ and $x = 2.5$.

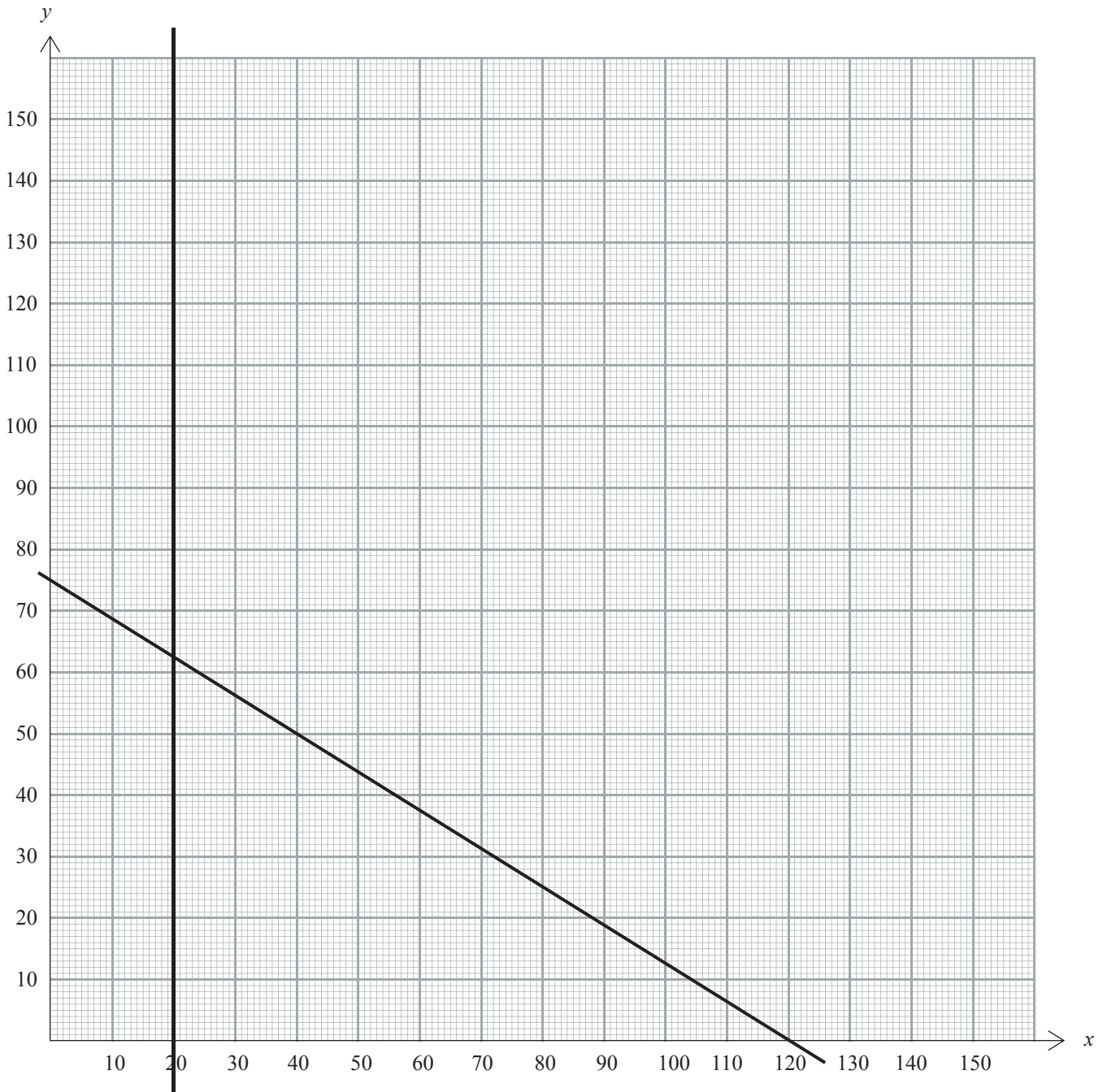
To investigate the Newton-Raphson method, the student uses an initial value of 2.5. Her first iteration gives an approximate value of 2.291000 for α .

The table shows the results for 20 iterations of both the bisection and Newton-Raphson methods.

Iteration Number	Bisection Method		Newton-Raphson Method
	x_1	x_2	
0	2.000000	3.000000	2.500000
1	2.000000	2.500000	2.291000
2	2.250000	2.500000	2.258803
3	2.250000	2.375000	2.258101
4	2.250000	2.312500	2.258101
5	2.250000	2.281250	2.258101
6	2.250000	2.265625	2.258101
7	2.257813	2.265625	2.258101
8	2.257813	2.261719	2.258101
9	2.257813	2.259766	2.258101
10	2.257813	2.258789	2.258101
11	2.257813	2.258301	2.258101
12	2.258057	2.258301	2.258101
13	2.258057	2.258179	2.258101
14	2.258057	2.258118	2.258101
15	2.258087	2.258118	2.258101
16	2.258087	2.258102	2.258101
17	2.258095	2.258102	2.258101
18	2.258099	2.258102	2.258101
19	2.258101	2.258102	2.258101
20	2.258101	2.258101	2.258101

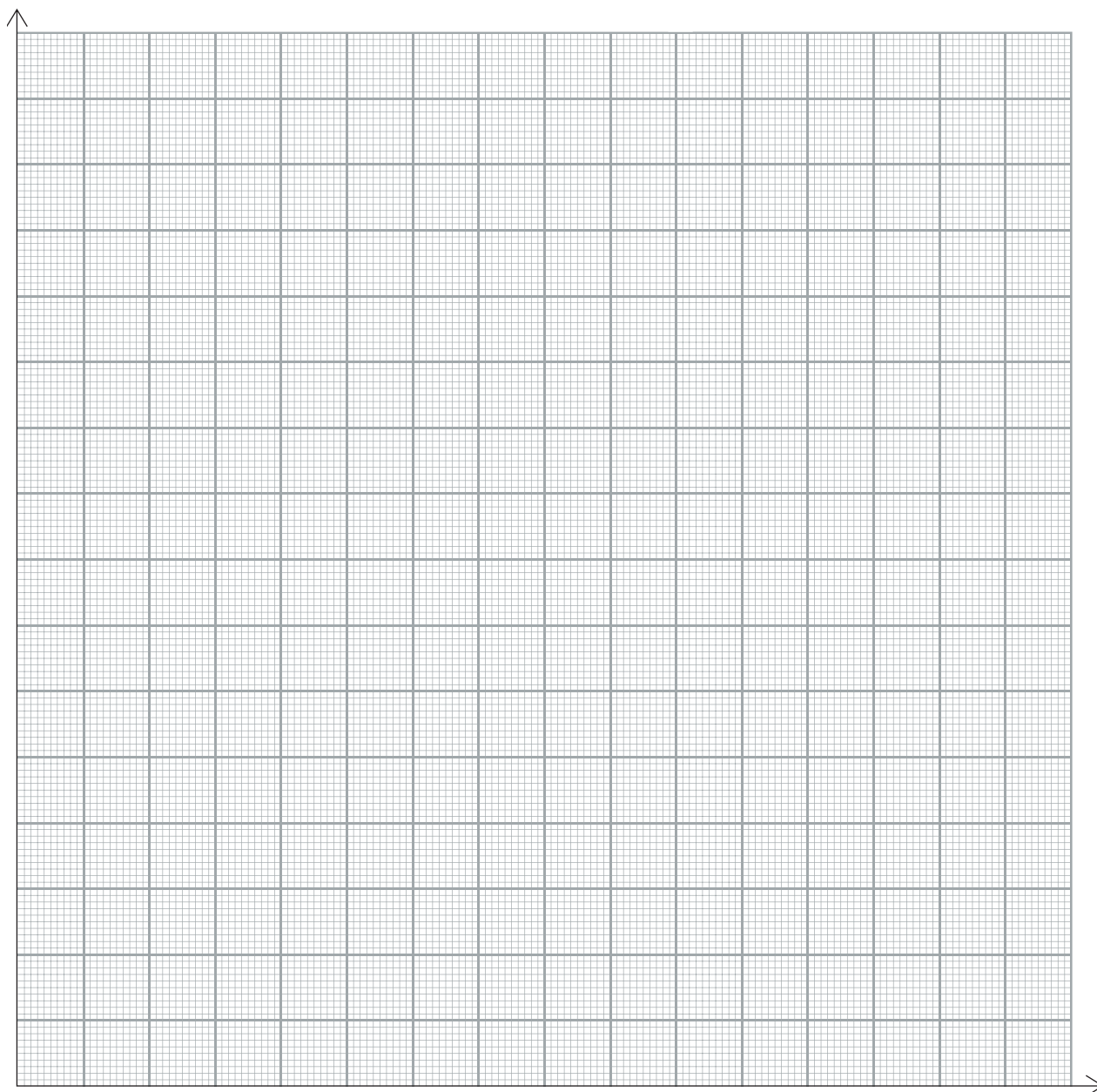
If you need to redraw your graph in Question Three, use this grid.

Assessor's
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If you wish to draw a graph as part of your answer to Question Four (b), use this grid.

Assessor's
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If you need to redraw your graph in Question Seven, use this grid.

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