For Supervisor's use only

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90644

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

Level 3 Statistics and Modelling, 2004

90644 Solve equations

Credits: Four
9.30 am Monday 15 November 2004

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Make sure you have a copy of the Formulae and Tables Booklet L3–STATF.

You should answer ALL the questions in this booklet.

If you need more space for any answer, use the pages provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–18 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Achievement Criteria	*For Assessor's use only*	
Achievement	**Achievement with Merit**	**Achievement with Excellence**
Solve equations. ☐	Solve problems involving equations. ☐	Analyse or interpret the outcome of the process used to solve the equations or linear programming problems. ☐
Overall Level of Performance ☐		

You are advised to spend 75 minutes answering the questions in this booklet.

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Show **ALL** working.

QUESTION ONE

(a) Solve the system of equations:

$$4x + y - 2z = -18$$

$$x + y + z = 0$$

$$-2x + 3z = 17$$

(b) Solve the system of equations:

$$a + 2b + 2c = 18$$

$$3a - 6b + 2c = 12$$

$$a + 6b - 3c = 13$$

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This page may be used for working, but **NO** work on this page will be marked.

QUESTION TWO

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The equation $x^4 - 6x + 1 = 0$ has a real root between 0 and 1.

Complete two iterations of either the bisection method, or the Newton-Raphson method, to find an approximation to this root. Show each iterate.

Use $x_0 = 0.5$ as the starting value if you use the Newton-Raphson method.

Use $x_0 = 0$ and $x_1 = 1$ as the starting values if you use the bisection method.

QUESTION THREE

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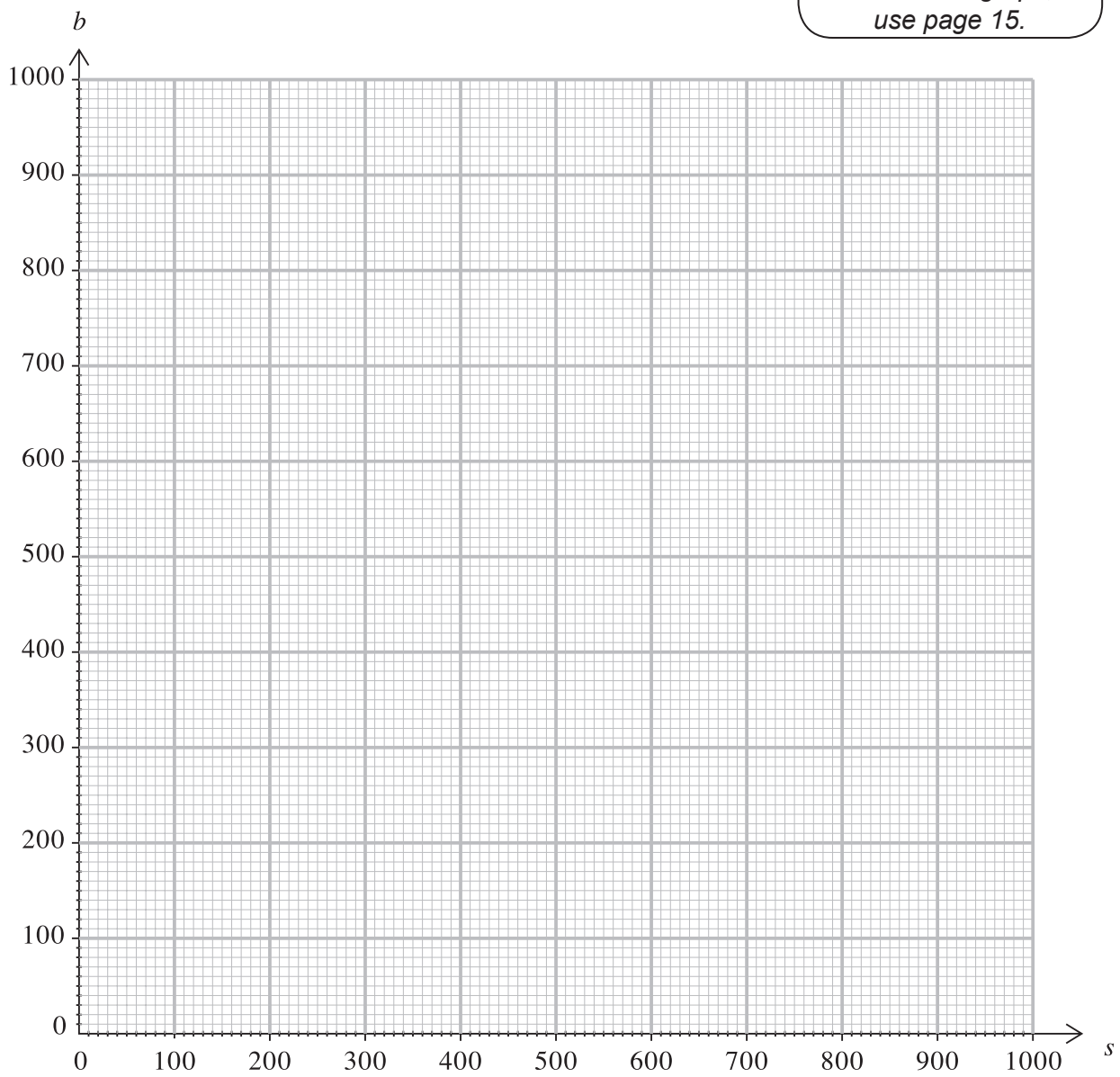
Andy's confectionery shop sells sweets.
Bags of wine gums and jaffas are put together in two different combinations.
Small bags are made up of 250 grams of wine gums and 250 grams of jaffas.
Big bags are made up of 250 grams of wine gums and 750 grams of jaffas.
Andy has 180 kilograms of wine gums and 240 kilograms of jaffas in stock.
Let the number of small bags Andy sells be s and the number of big bags he sells be b.

A linear programming problem for this situation has the following constraints:

$250s + 250b \leq 180\,000$
$250s + 750b \leq 240\,000$
$s \geq 0$
$b \geq 0$

(a) Graph the constraints given above and indicate the feasible region.

If you need to redraw this graph, use page 15.



Assessor's use only

(b) Andy sells small bags for \$3 each and big bags for \$5 each.
His revenue R (in dollars) is given by $R = 3s + 5b$.

(i) Determine how many of each size bag Andy should sell to maximise his revenue.

(ii) State Andy's maximum revenue.

QUESTION FOUR

Assessor's use only

Andy has decided to make up bags containing 200 grams of coloured sweets.
The bags contain different mixtures of coloured sweets.
Andy has three different colours of sweets in stock.

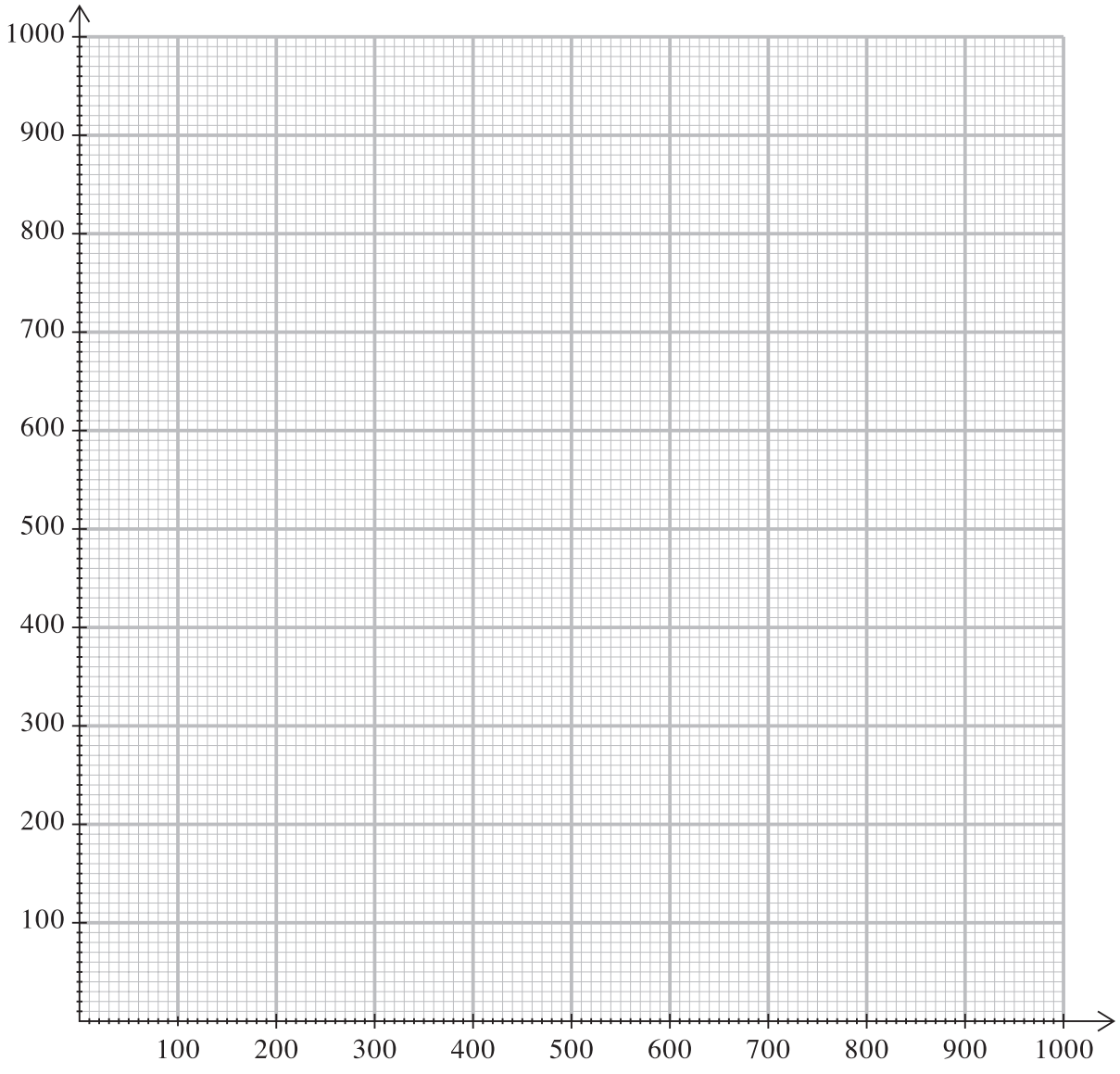
Bags of mixture A contain 100 grams of purple sweets, 50 grams of red sweets and 50 grams of green sweets.
Bags of mixture B contain 100 grams of purple sweets and 100 grams of green sweets.

Andy has a total of 60 kilograms of purple sweets, 25 kilograms of red sweets and 45 kilograms of green sweets.

Andy sells bags of mixture A for $2.45, and bags of mixture B for $1.95.

You may use the axes below to help you answer this question.

If you need to redraw this graph, use page 16.



Assessor's use only

(a) Determine how many bags of each mixture Andy should sell to maximise his revenue, and clearly state his maximum revenue.

(b) Andy later decides to sell both mixtures for the same price.

Explain fully the effect this decision has on the number of bags of each mixture that he should sell to maximise his revenue.

QUESTION FIVE

Assessor's use only

(a) Only three companies sell licensed Olympic merchandise in Athenville.

In a particular week a total of $100 000 worth of the merchandise is sold. Company A sells $1 600 more than Company B and Company C combined. Company B sells twice as much as Company C.

Set up and solve a system of equations to find the value of the merchandise each of the three companies has sold in this particular week.

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(b) *Burger Queen* has produced three new burgers.
Information relating to these burgers is given in the table below.

Name of Burger	Surfer Chicken	Beef Party	Fish'n Time
Type of filling	Chicken	Beef	Fish
Cost to buy	$2.50	$3.00	$2.00
Weight of filling	150 g	200 g	120 g

Last week a total of 740 of the new burgers were sold. The total weight of the fillings for the three new burgers that week was 122.5 kg. The total income from the three new burgers that week was $1935.

Set up and solve a system of equations to find the number of each type of new burger sold that week by *Burger Queen*.

QUESTION SIX

Assessor's use only

A manufacturing business has just bought a new machine and is investigating how long it might be until the machine needs to be replaced.

A function that estimates the amount of money that the machine earns is $E = 250e^{-0.2t}$ where E is the amount the machine earns (in thousands of dollars) and t is the time in years since the machine was obtained.

A function that estimates the cost of repairs and maintenance for the machine is $C = 0.3t^2$ where C is the cost (in thousands of dollars) and t is the age of the machine in years.

To estimate when the earnings from the machine will equal the costs of repairs and maintenance for the machine, the following equation must be solved:

$$250e^{-0.2t} - 0.3t^2 = 0$$

It is known that the solution for this equation is a t value that is greater than 10 years.

Use either the bisection method, or the Newton-Raphson method, to find correct to one decimal place, how many years it will be until the machine needs to be replaced. Show each iterate.

Note: If $f(t) = 250e^{-0.2t} - 0.3t^2$

then $f'(t) = -50e^{-0.2t} - 0.6t$.

QUESTION SEVEN

Assessor's use only

The following set of equations does not have a unique solution.

$$-2x + 4y - 2z = 11$$

$$3x - 5y + 2z = -3$$

$$-x + 2y - z = 7$$

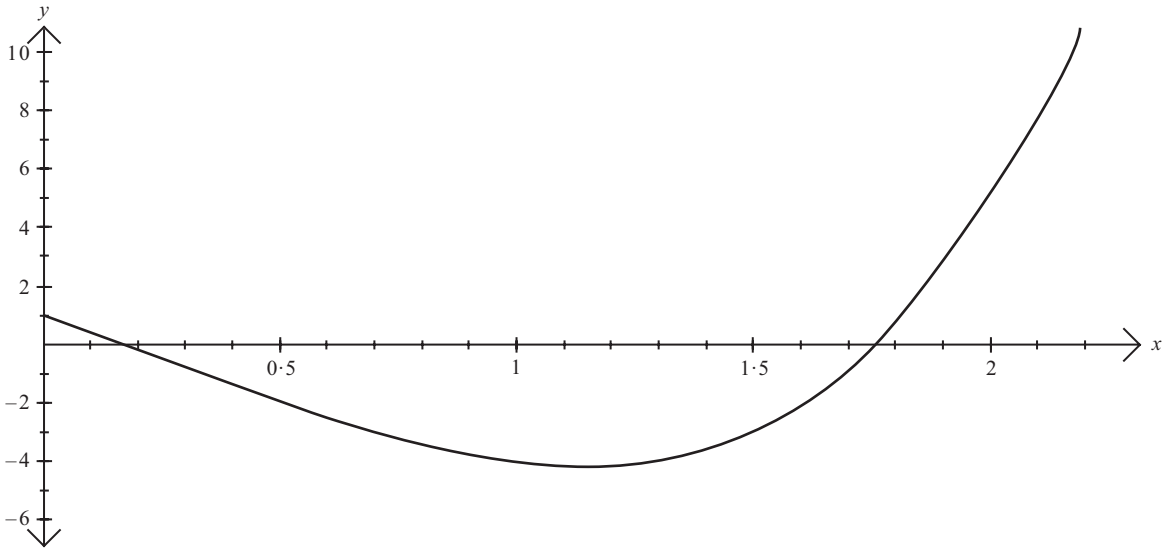
Give a geometrical description of how these three planes relate to each other.

QUESTION EIGHT

Assessor's use only

In Question Two you were asked to find an approximation to the root of $x^4 - 6x + 1 = 0$ that lies between 0 and 1.

When answering this question using the Newton-Raphson method, a student started with an initial value $x_0 = 1.5$ rather than $x_0 = 0.5$.



Part of the graph of $y = x^4 - 6x + 1$ is shown above.

(a) Use the graph to show geometrically how the first iterate x_1, and the second iterate x_2, are found using the Newton-Raphson method. Clearly show x_1 and x_2 on the graph above.

(b) Explain why using an initial value of $x_0 = 1.5$ will not result in a sequence of iterates that converges to the root of $x^4 - 6x + 1 = 0$ that lies between 0 and 1.

If you have made a mistake and need to redraw a graph, use the appropriate copy printed here.

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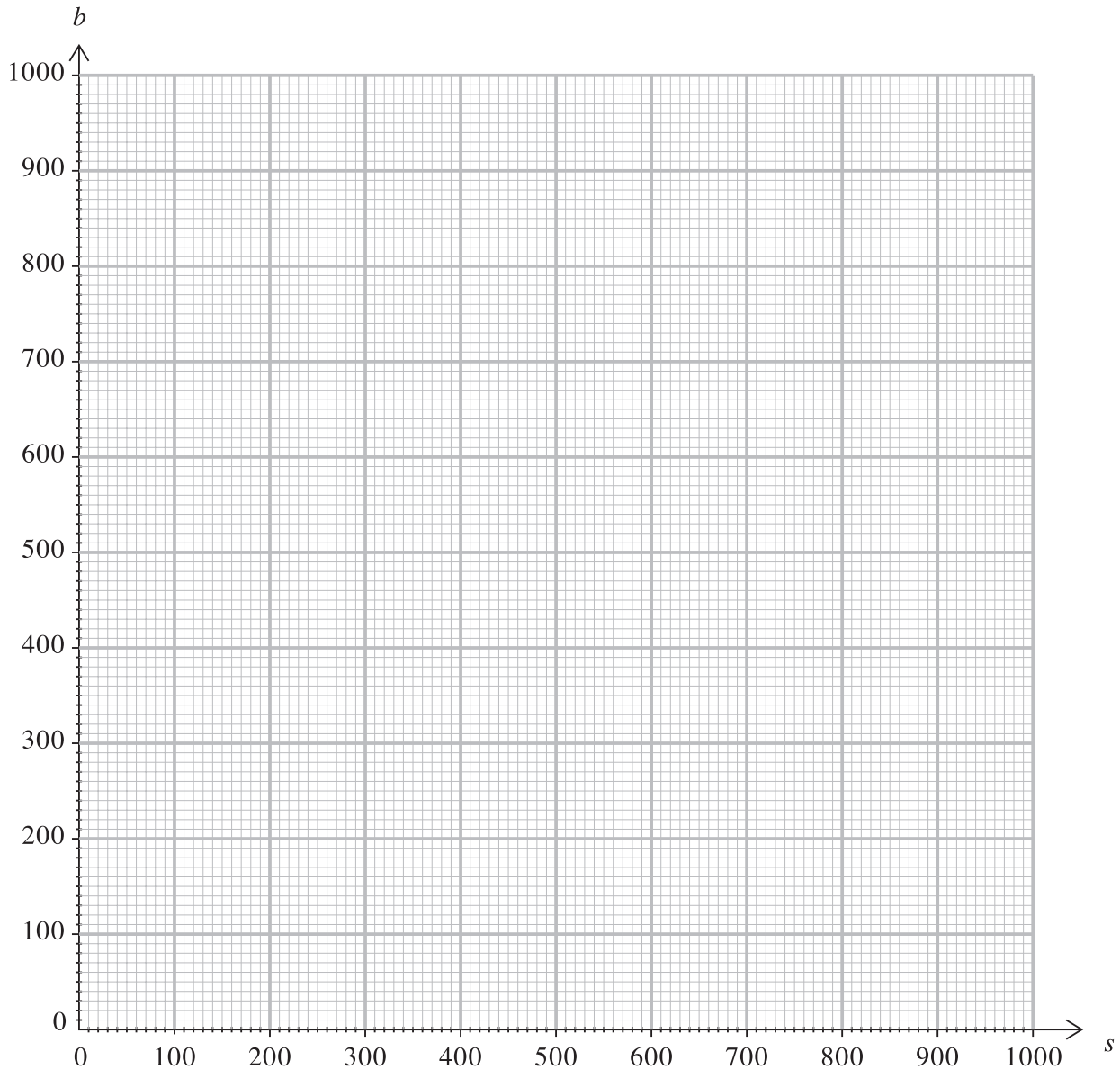

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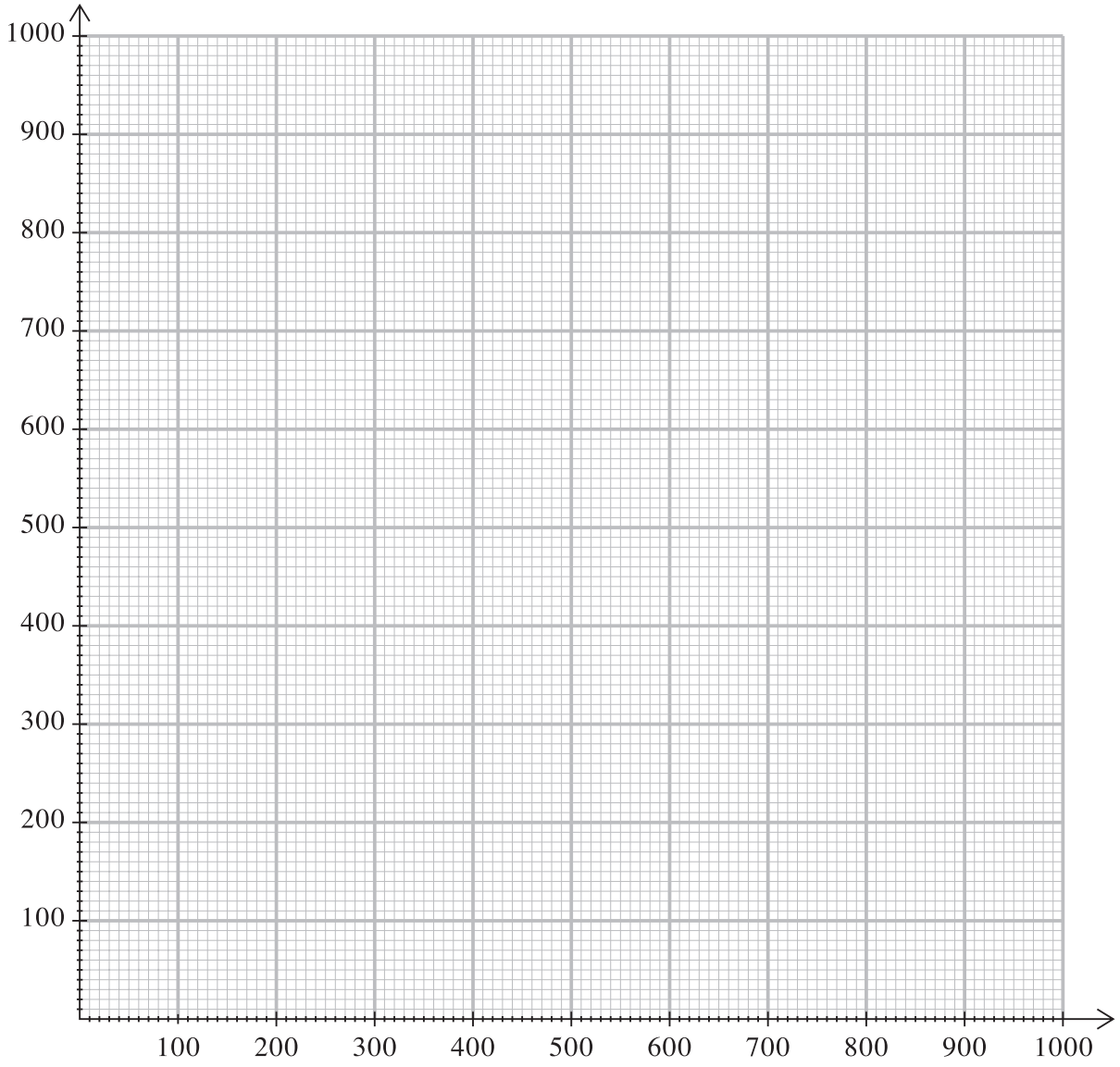

Extra paper for continuation of answers if required.
Clearly number the question.

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