90644

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

Level 3 Statistics and Modelling, 2003

90644 Solve equations.

Credits: Four

Answer ALL questions in the spaces provided in this booklet.

Show ALL working for ALL questions.

Check that this booklet has pages 2–9 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

For Assessor's use only **Achievement Criteria**		
Achievement	**Achievement with Merit**	**Achievement with Excellence**
Solve equations.	Solve problems involving equations.	Analyse or interpret the outcome of the process used to solve the equations or linear programming problems.
Overall Level of Performance		

You are advised to spend 50 minutes answering the questions in this booklet.

Show ALL working.

QUESTION ONE

(a) Solve the system of equations:

$$\begin{aligned} 3x + 2y - z &= 12 \\ x - 6y + 3z &= -6 \\ 2x - 4y - 5z &= 14 \end{aligned}$$

(b) Solve the system of equations:

$$\begin{aligned} 3x - y + 2z &= 16 \\ x - 2y &= 7 \\ 5x + 3y - 2z &= -14 \end{aligned}$$

QUESTION TWO

Bill plans to develop some of his land in macadamia nuts and chestnuts.

He has a number of constraints on the areas of the crops he can plant.

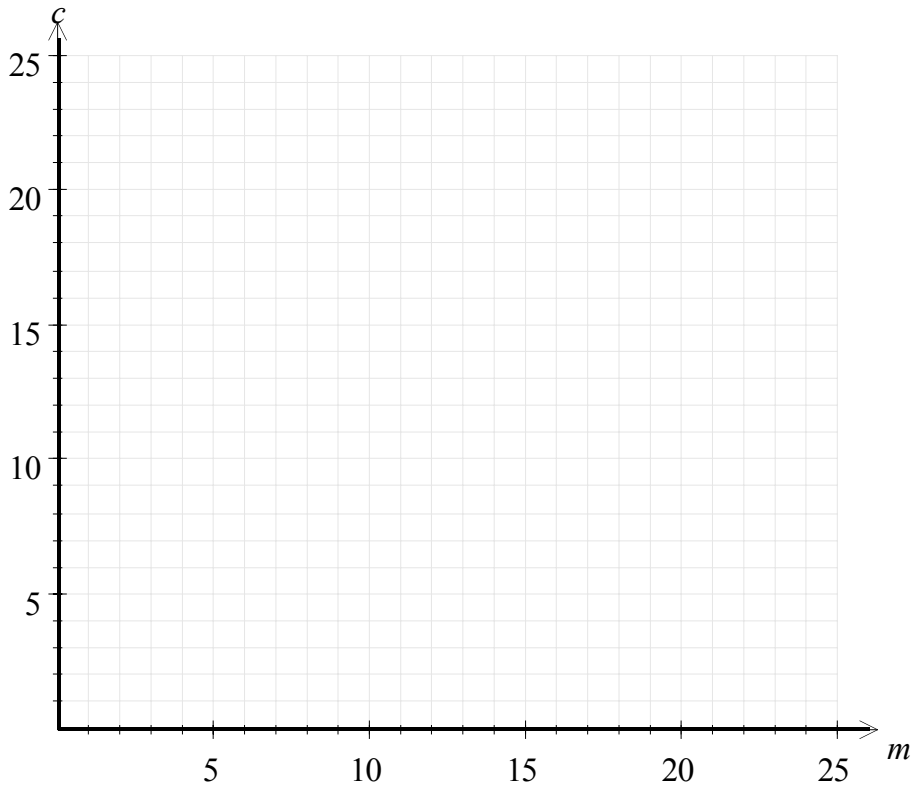
Suppose he plants m hectares in macadamia nuts and c hectares in chestnuts.

(a) Draw the following five constraints on the axes provided below, and indicate the feasible region.

$m + c \leq 15$ $\quad$ $c \geq 6$

$4m + c \leq 24$ $\quad$ $m \geq 2$

$c \geq 2m$



(b) Bill can estimate his return, in dollars, using the function $R = 54\,000m + 36\,000c$.

Calculate his maximum estimated return. You may read the coordinates of the vertices of the feasible region directly from the graph.

QUESTION THREE

The (real) cube root of 20 lies between 2 and 3.

Its value can be found by solving the equation $x^3 - 20 = 0$.

Find an approximation to the cube root of 20

either by obtaining three iterates using the Newton-Raphson method with $x = 2$ as the starting value

or by obtaining three iterates using the bisection method with $x = 2$ and $x = 3$ as the starting values.

Show all steps in your working clearly.

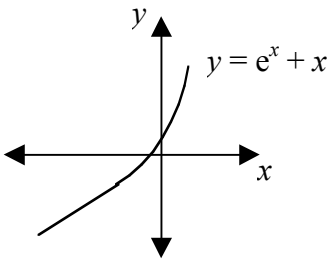
QUESTION FOUR

The equation $e^x + x = 0$ has one root.

Use **either** the Newton-Raphson method **or** the bisection method to find an approximation to this root, correct to four decimal places.

Show all steps in your working clearly.

Note: If $f(x) = e^x + x$, then $f'(x) = e^x + 1$.



QUESTION FIVE

Three brands of canned mixed vegetables contain the following quantities of protein, carbohydrates, and minerals.

Brands	Protein	Carbohydrates	Minerals
All Veges	4 g	2 g	3 g
Betta Veges	2 g	3 g	3 g
Canned Veges	4 g	4 g	2 g

Set up and solve a system of equations to find how many cans of each brand should be bought, so the total quantity of each of protein, carbohydrates and minerals is 100 g?

QUESTION SIX

In a card game, players take turns to pick up and put down cards.

Each card held in a particular suit scores a certain number of penalty points depending on the suit (hearts, spades or diamonds – clubs do not have any penalty points).

A player's penalty is found by adding the number of points for the cards held.

A particular game ended as follows:

- player A held 2 hearts, 1 diamond and 3 spades and has 34 penalty points
- player B held 1 heart, 4 diamonds and 5 spades and has 45 penalty points
- player C held 3 hearts, 2 diamonds and 1 spade and has 43 penalty points.

Set up and solve a system of equations to find the number of penalty points the cards in each suit receives.

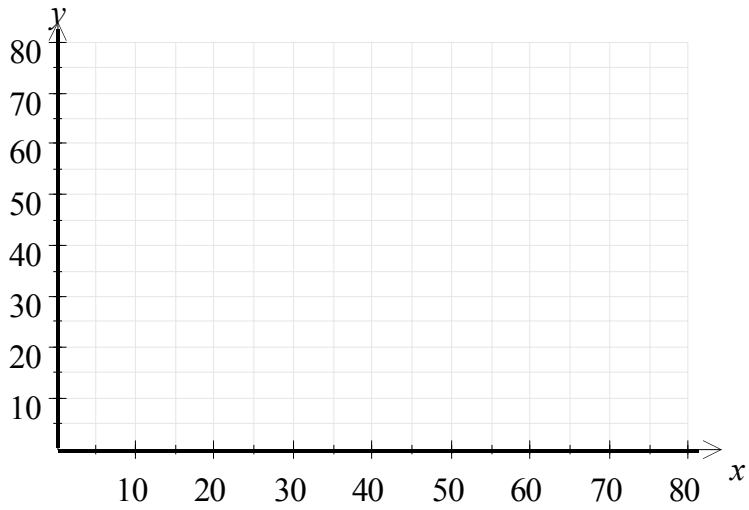
QUESTION SEVEN

The owner of NZJEANS sells only $\mathbf{X}_{\mathbf{vi}}$ and $\mathbf{Y}_{\mathbf{co}}$ jeans.

He buys no more than 60 pairs per week and sells all he buys.

(a) A pair of $\mathbf{X}_{\mathbf{vi}}$ jeans costs the owner \$60, a pair of $\mathbf{Y}_{\mathbf{co}}$ jeans costs the owner \$120 and he spends a maximum of \$4800 per week buying jeans.

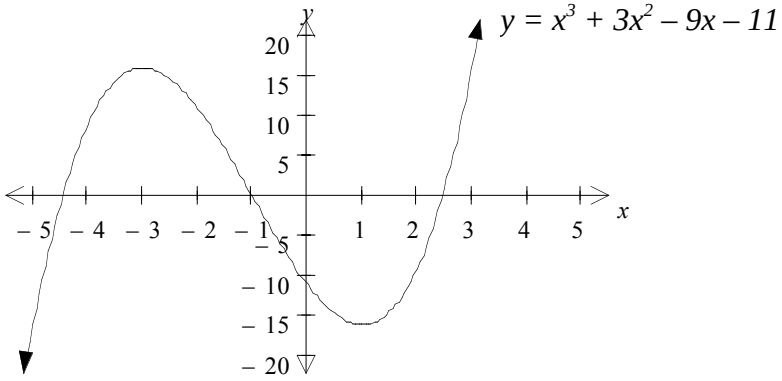
If the profit for a pair of $\mathbf{X}_{\mathbf{vi}}$ jeans is \$10 and the profit for a pair of $\mathbf{Y}_{\mathbf{co}}$ jeans is \$12, how many of each type of jeans should he buy in order to make the biggest profit, and what is that profit?



(b) If he now decides to sell each pair of $\mathbf{Y_{co}}$ jeans for a profit of $20, how many of each type of jeans should he buy to make the biggest profit? Explain what effect the selling price of $\mathbf{Y_{co}}$ jeans has on the situation.

QUESTION EIGHT

Elizabeth wants to find the positive root to the equation $x^3 + 3x^2 - 9x - 11 = 0$ using the Newton-Raphson method.



Explain why using a starting value of $x = 1$ is not a good choice.

QUESTION NINE

Attempt to solve the system of equations

$$\begin{aligned} 2x - y + 3z &= 17 \\ x + 2y - z &= -4 \\ 3x + y + 2z &= 10 \end{aligned}$$

and explain the nature of the equations.

Give a geometrical interpretation of the result.

Assessment Schedule (Sample)

Statistics and Modelling: Solve equations (90644)

	Achievement Criteria	Qn No.	Evidence	Code	Judgement	Sufficiency
ACHIEVEMENT	Solve equations.	1 (a)	Eliminate variables successively to find solution $x = 3$ $y = \frac{1}{2}$ (or 0.5) $z = -2$	A1	No alternative.	***Achievement:*** Three of Code A including at least One Code A1 **and** One Code A2
		1(b)	Eliminate variables successively to find solution $x = 1$ $y = -3$ $z = 5$	A1	No alternative.	
		2 (b)	Vertex $R = 54\,000m + 36\,000c$ (2,6) 324 000 (3,6) 378 000 (4,8) 504 000 (3,12) 594 000 (2,13) 576 000 maximum return, R = \$594 000 when $m = 3$ and $c = 12$.	A2	Maximum value of R is stated.	
		3	Newton-Raphson method with starting value 2 gives solution 2.7147. (iterates 3 2.7407407 2.7146696) Bisection method with starting values [2,3] gives solution 2.625 **or** in the interval [2.5,2.75] (iterates [intervals] 2.5 [2,3] 2.75 [2.5,3] 2.625 [2.5,2.75])	A2	No penalty for extra iterates. For bisection method, solution can be expressed as an interval or a value.	

	Achievement Criteria	No.	Evidence	Code	Judgement	Sufficiency
ACHIEVEMENT WITH MERIT	Solve problems involving equations.	4	Newton-Raphson method, with starting value 0, gives iterates -0.5 -0.566311 -0.567143 -0.567143 Newton-Raphson method solution: -0.5671 Bisection method also gives solution of -0.5671	 A2 M	 Ignore one minor error in arithmetic. Accept other starting values.	***Merit:*** Achievement plus **Three Code M** **or** **Four Code M**
		5	Form the system of equations $4a + 2b + 4c = 100$ $2a + 3b + 4c = 100$ $3a + 3b + 2c = 100$ and solve to obtain $a = 10,\ b = 20,\ c = 5$	 A1 M	Systems of equations and solutions required. No alternative.	
		6	Form the system of equations $2h + d + 3s = 34$ $h + 4d + 5s = 45$ $3h + 2d + s = 43$ and solve to obtain $h = 10,\ d = 5,\ s = 3$	 A1 M	Systems of equations and solutions required. No alternative.	
		7 (a)	x = number of $\mathbf{X_{vi}}$ bought y = number of $\mathbf{Y_{co}}$ bought The constraints are: $x + y \le 60$ $60x + 120y \le 4800$ x and y non-negative Objective function: $R = 10x + 12y$ Vertex $R = 10x + 12y$ (0,0) 0 (60,0) 600 (40,20) 640 (0,40) 480 maximum return, R = \$640 when $\mathbf{X_{vi}} = 40$ and $\mathbf{Y_{co}} = 20$.	 A2 M	Constraints and objective function must be given. Must state the number of each type of jeans that gives the maximum and the maximum value.	

	Achievement Criteria	No.	Evidence	Code	Judgement	Sufficiency
ACHIEVEMENT WITH EXCELLENCE	Analyse or interpret the outcome of the process used to solve equations or linear programming problems.	7 (b)	New objective function: $R = 10x + 20y$ Vertex | $R = 10x + 20y$ (0, 0) (60, 0) (40, 20) (0, 40) | 800 maximum return, R = \$800 when you choose any integer (x, y) with $0 \le x \le 40$ and $x + 2y = 80$. Multiple solutions because the objective function and a constraint have the same slope.	E	Needs to indicate the concept of multiple solutions.	***Excellence:*** Merit **plus** Two of Code E
		8	The gradient of the tangent to the curve at $x = 1$ is zero, so the next iterate is undefined.	E		
		9	Equations are inconsistent. System represents three planes that are not parallel but do not have a unique point of intersection (planes represent the faces of a triangular prism, or any two planes intersect in a line that is parallel to the third plane).	E	Must show attempt to solve equations resulting in two equations of the type $ax + by = c$ $ax + by = d$ that are clearly inconsistent. Conclusion could be expressed as any equivalent statement.	