

90643



NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA



National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

Level 3 Statistics and Modelling, 2003

90643 Solve straightforward problems involving probability.

Credits: Four

Answer ALL questions in the spaces provided in this booklet.

Show ALL working for ALL questions.

Check that this booklet has pages 2–6 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

| <i>For Assessor's use only</i> | | |
|-------------------------------------------------------|-------------------------------|------------------------------------|
| Achievement Criteria | | |
| Achievement | Achievement with Merit | Achievement with Excellence |
| Solve straightforward problems involving probability. | Solve probability problems. | Apply probability theory. |
| Overall Level of Performance | | |

You are advised to spend 50 minutes answering the questions in this booklet.

Show ALL working.

QUESTION ONE

The number of newspapers (Y) purchased by households in a provincial town each week has the following probability distribution:

| | | | | | | | |
|-----------|------|------|------|------|------|------|------|
| y | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| P (Y = y) | 0.19 | 0.15 | 0.12 | 0.04 | 0.06 | 0.02 | 0.42 |

- (a) Calculate the probability that at least two newspapers are purchased each week by a household that is chosen at random.

- (b) Calculate the variance of Y.

QUESTION TWO

A survey of newspaper purchasing patterns in a particular region found that $\frac{3}{5}$ of the households surveyed purchased a daily newspaper during the previous week. The survey also found that in $\frac{7}{10}$ of the households where a daily newspaper was purchased during the previous week, a Sunday paper was also purchased. A Sunday paper was purchased in $\frac{1}{4}$ of households where no daily newspaper was purchased.

You may wish to draw a probability tree in the space below.

What is the probability that, in a household chosen at random from those surveyed:

- (a) a daily newspaper was purchased **or** a Sunday newspaper was purchased (**but not both**)?

- (b) a daily newspaper was purchased, given that a Sunday newspaper was purchased?

QUESTION THREE

In a different survey of 120 households, where 70 of them purchased a daily newspaper, the following information was obtained.

| Nature of household | Number of households | Number of households purchasing a daily newspaper |
|------------------------------------------------|----------------------|---------------------------------------------------|
| Contains at least one primary school student | 44 | 27 |
| Contains at least one secondary school student | 39 | 21 |
| Contains no school age children | 63 | 36 |

Households may contain both primary and secondary students.

- (a) Find the probability that a household chosen at random from this sample contains both primary and secondary school students.

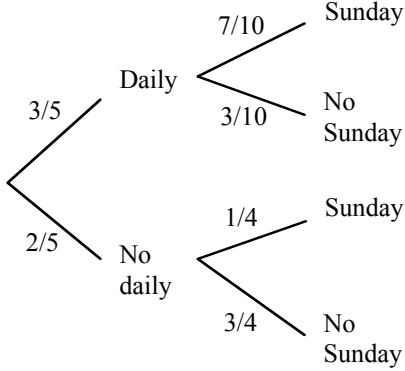
A household is randomly selected.

- (b) Demonstrate mathematically that the event “the household contains both primary and secondary school students” and the event “the household purchases a daily newspaper” are not independent.

- (c) Find the probability that a household contains at least one secondary school student, given that the household purchases at least one daily newspaper.

Assessment Schedule (sample)

Statistics and Modelling: Solve straightforward problems involving probability (90643)

| | Achievement Criteria | Qn No. | Evidence | Code | Judgement | Sufficiency |
|--------------------|-------------------------------------------------------|--------|-------------------------------------------------------------------------------------------------------------------------|------|---------------|----------------------------------------|
| ACHIEVEMENT | Solve straightforward problems involving probability. | 1(a) | $P(Y \geq 2)$ $= 1 - (0.19 + 0.15)$ $= 0.66$ | A | Or equivalent | <i>Achievement:</i> Three of Code A |
| | | 1(b) | $E(Y) = 3.37$ $E(Y^2) = 17.57$ $\text{Var}(Y) = 17.57 - 3.37^2$ $= 6.21$ | A | Or equivalent | |
| | | 2 |  | | | |
| | | 2(a) | $P(D \text{ or } S)$ $= \frac{3}{5} \times \frac{3}{10} + \frac{2}{5} \times \frac{1}{4}$ $= \frac{7}{25} \quad (0.28)$ | A | Or equivalent | |
| | | 3(a) | $P(P\&S)$ $= \frac{26}{120} \quad (0.217)$ | A | Or equivalent | |

| | Achievement Criteria | Qn No. | Evidence | Code | Judgement | Sufficiency |
|------------------------|-----------------------------|--------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|---------------|-----------------------------------------------------------------------------------------------|
| ACHIEVEMENT WITH MERIT | Solve probability problems. | 2(b) | $P(D S)$ $= \frac{P(D \cap S)}{P(S)}$ $= \frac{21}{50} \div \frac{13}{25}$ $= \frac{21}{26} \quad (0.808)$ | A M | Or equivalent | Merit: Achievement plus Three of Code M or Four of Code M |
| | | 3(b) | $P(P \& S) = \frac{26}{120} \quad (0.217)$ $P(D) = \frac{70}{120} \quad (0.583)$ $P(P \& S \cap D)$ $= \frac{14}{120} \quad (0.117)$ $P(P \& S) \times P(D)$ $= \frac{91}{720} \quad (0.126)$ <p>As $P(P \& S \cap D) \neq P(P \& S) \times P(D)$ then not independent events.</p> | A M | Or equivalent | |
| | | 3(c) | $P(S D)$ $= \frac{P(S \cap D)}{P(D)}$ $= \frac{21}{120} \div \frac{70}{120}$ $= \frac{3}{10} \quad (0.3)$ | A M | Or equivalent | |
| | | 4 | <p>P(4 boys and 2 girls)</p> $= \frac{{}^{10}C_4 \times {}^8C_2}{{}^{18}C_6}$ $= 0.317$ | A M | Or equivalent | |

| | Achievement Criteria | Qn No. | Evidence | Code | Judgement | Sufficiency | | | | | | | | | | | | |
|------------------------------------|---------------------------|---------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|---------------|-------------|-----|-----|-----|----------|---------------|---------------|---------------|-----|---------------|---|--|-------------------------------------------------------------------|
| ACHIEVEMENT WITH EXCELLENCE | Apply probability theory. | 5 | <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="text-align: center;">x</td> <td style="text-align: center;">1</td> <td style="text-align: center;">2</td> <td style="text-align: center;">3</td> <td style="text-align: center;">...</td> <td style="text-align: center;">k</td> </tr> <tr> <td style="text-align: center;">$P(X=x)$</td> <td style="text-align: center;">$\frac{1}{k}$</td> <td style="text-align: center;">$\frac{1}{k}$</td> <td style="text-align: center;">$\frac{1}{k}$</td> <td style="text-align: center;">...</td> <td style="text-align: center;">$\frac{1}{k}$</td> </tr> </table> $E(X) = 1 \times \frac{1}{k} + 2 \times \frac{1}{k} + \dots + k \times \frac{1}{k}$ $= (1 + 2 + 3 + \dots + k) \frac{1}{k}$ $= \frac{k(k+1)}{2} \times \frac{1}{k}$ $= \frac{k+1}{2}$ $E(X^2) = 1^2 \times \frac{1}{k} + 2^2 \times \frac{1}{k} + \dots + k^2 \times \frac{1}{k}$ $= (1^2 + 2^2 + \dots + k^2) \frac{1}{k}$ $= \frac{k(k+1)(2k+1)}{6} \times \frac{1}{k}$ $= \frac{(k+1)(2k+1)}{6}$ $\text{Var}(X) = E(X^2) - [E(X)]^2$ $= \frac{(k+1)(2k+1)}{6} - \frac{(k+1)^2}{4}$ $= \frac{k^2 - 1}{12}$ | x | 1 | 2 | 3 | ... | k | $P(X=x)$ | $\frac{1}{k}$ | $\frac{1}{k}$ | $\frac{1}{k}$ | ... | $\frac{1}{k}$ | A | | <p>Excellence:</p> <p>Merit plus Code E</p> |
| | | | x | 1 | 2 | 3 | ... | k | | | | | | | | | | |
| $P(X=x)$ | $\frac{1}{k}$ | $\frac{1}{k}$ | $\frac{1}{k}$ | ... | $\frac{1}{k}$ | | | | | | | | | | | | | |
| A E | Or equivalent. | | | | | | | | | | | | | | | | | |