

L3-CALCF



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National Certificate of Educational Achievement
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Level 3 Calculus, 2005

FORMULAE AND TABLES BOOKLET

This booklet is for use with Calculus 90635, 90636, 90638 and 90639.

Check that this booklet has pages 2–4 in the correct order and that none of these pages is blank.

YOU MAY KEEP THIS BOOKLET AT THE END OF THE EXAMINATION.

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CALCULUS – USEFUL FORMULAE

ALGEBRA

Quadratics

If $ax^2 + bx + c = 0$

$$\text{then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Logarithms

$$y = \log_b x \Leftrightarrow x = b^y$$

$$\log_b(xy) = \log_b x + \log_b y$$

$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$

$$\log_b(x^n) = n \log_b x$$

$$\log_b x = \frac{\log_a x}{\log_a b}$$

Complex Numbers

$$\begin{aligned} z &= x + iy \\ &= r \operatorname{cis} \theta \\ &= r(\cos \theta + i \sin \theta) \end{aligned}$$

$$\begin{aligned} \bar{z} &= x - iy \\ &= r \operatorname{cis}(-\theta) \\ &= r(\cos \theta - i \sin \theta) \end{aligned}$$

$$r = |z| = \sqrt{z\bar{z}} = \sqrt{(x^2 + y^2)}$$

$$\theta = \arg z$$

$$\text{where } \cos \theta = \frac{x}{r}$$

$$\text{and } \sin \theta = \frac{y}{r}$$

De Moivre's Theorem:

If n is any integer then

$$(r \operatorname{cis} \theta)^n = r^n \operatorname{cis}(n\theta)$$

COORDINATE GEOMETRY

Straight Line

$$\text{Equation } y - y_1 = m(x - x_1)$$

Circle

$$(x - a)^2 + (y - b)^2 = r^2$$

has a centre (a, b) and radius r

Parabola

$$y^2 = 4ax \text{ or } (at^2, 2at)$$

Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ or } (a \cos \theta, b \sin \theta)$$

Hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \text{ or } (a \sec \theta, b \tan \theta)$$

$$\text{asymptotes } y = \pm \frac{b}{a} x$$

CALCULUS

Differentiation

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
$\ln x$	$\frac{1}{x}$
e^{ax}	ae^{ax}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\cot x$	$-\operatorname{cosec}^2 x$

Product rule

$$(f \cdot g)' = f \cdot g' + g \cdot f' \quad \text{or if } y = uv \text{ then } \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

Quotient rule

$$\left(\frac{f}{g}\right)' = \frac{g \cdot f' - f \cdot g'}{g^2} \quad \text{or if } y = \frac{u}{v} \text{ then } \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Integration

$f(x)$	$\int f(x) \, dx$
x^n	$\frac{x^{n+1}}{n+1} + c$
$\frac{1}{x}$	$\ln x + c$
$\frac{f'(x)}{f(x)}$	$\ln f(x) + c$

Composite Function or Chain rule

$$(f(g))' = f'(g) \cdot g'$$

$$\text{or if } y = f(u) \text{ and } u = g(x) \text{ then } \frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Volume of Revolution

$$y = f(x) \text{ between } x = a \text{ and } x = b$$

rotated about the x -axis

$$\text{Volume} = \int_a^b \pi y^2 \, dx$$

NUMERICAL METHODS

Trapezium Rule

$$\int_a^b f(x) \, dx \approx \frac{1}{2} h [y_0 + y_n + 2(y_1 + y_2 + \dots + y_{n-1})]$$

$$\text{where } h = \frac{b-a}{n} \text{ and } y_r = f(x_r)$$

Simpson's Rule

$$\int_a^b f(x) \, dx \approx \frac{1}{3} h [y_0 + y_n + 4(y_1 + y_3 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2})]$$

$$\text{where } h = \frac{b-a}{n}, y_r = f(x_r) \text{ and } n \text{ is even.}$$

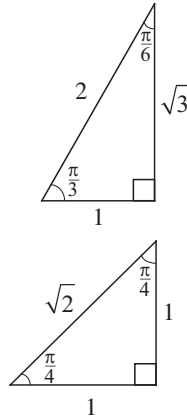
TRIGONOMETRY

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

**Sine Rule**

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine Rule

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Identities

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \operatorname{cosec}^2 \theta$$

General Solutions

$$\text{If } \sin \theta = \sin \alpha \text{ then } \theta = n\pi + (-1)^n \alpha$$

$$\text{If } \cos \theta = \cos \alpha \text{ then } \theta = 2n\pi \pm \alpha$$

$$\text{If } \tan \theta = \tan \alpha \text{ then } \theta = n\pi + \alpha$$

where n is any integer

Compound Angles

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Double Angles

$$\sin 2A = 2 \sin A \cos A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2 \cos^2 A - 1 \\ &= 1 - 2 \sin^2 A \end{aligned}$$

Products

$$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$2 \cos A \sin B = \sin(A+B) - \sin(A-B)$$

$$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$$

Sums

$$\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}$$

$$\cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\cos C - \cos D = -2 \sin \frac{C+D}{2} \sin \frac{C-D}{2}$$

MEASUREMENT**Triangle**

$$\text{Area} = \frac{1}{2} ab \sin C$$

Trapezium

$$\text{Area} = \frac{1}{2} (a+b)h$$

Sector

$$\text{Area} = \frac{1}{2} r^2 \theta$$

$$\text{Arc length} = r\theta$$

Cylinder

$$\text{Volume} = \pi r^2 h$$

$$\text{Curved surface area} = 2\pi r h$$

Cone

$$\text{Volume} = \frac{1}{3} \pi r^2 h$$

$$\text{Curved surface area} = \pi r l \text{ where } l = \text{slant height}$$

Sphere

$$\text{Volume} = \frac{4}{3} \pi r^3$$

$$\text{Surface area} = 4\pi r^2$$