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## MATHEMATICS WITH CALCULUS: 2004

FORMULAE AND TABLES BOOKLET

Check that this booklet has pages 2–4 in the correct order and that none of these pages is blank

**YOU MAY KEEP THIS FORMULAE AND TABLES BOOKLET AT THE END OF THE EXAMINATION.**

# MATHEMATICS WITH CALCULUS - USEFUL FORMULAE

## ALGEBRA

### Quadratics

$$\text{If } ax^2 + bx + c = 0$$

$$\text{then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

### Logarithms

$$y = \log_b x \Leftrightarrow x = b^y$$

$$\log_b(xy) = \log_b x + \log_b y$$

$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$

$$\log_b(x^n) = n \log_b x$$

$$\log_b x = \frac{\log_a x}{\log_a b}$$

## Complex Numbers

$$\begin{aligned} z &= x + iy \\ &= r \operatorname{cis} \theta \\ &= r(\cos \theta + i \sin \theta) \end{aligned}$$

$$\begin{aligned} \bar{z} &= x - iy \\ &= r \operatorname{cis}(-\theta) \\ &= r(\cos \theta - i \sin \theta) \end{aligned}$$

$$r = |z| = \sqrt{z\bar{z}} = \sqrt{(x^2 + y^2)}$$

$$\theta = \arg z$$

$$\text{where } \cos \theta = \frac{x}{r}$$

$$\text{and } \sin \theta = \frac{y}{r}$$

De Moivre's Theorem:

If  $n$  is any integer then

$$(r \operatorname{cis} \theta)^n = r^n \operatorname{cis}(n\theta)$$

## COORDINATE GEOMETRY

### Straight Line

$$\text{Equation } y - y_1 = m(x - x_1)$$

### Circle

$$(x - a)^2 + (y - b)^2 = r^2$$

has centre  $(a, b)$  and radius  $r$

### Parabola

$$y^2 = 4ax \text{ or } (at^2, 2at)$$

### Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ or } (a \cos \theta, b \sin \theta)$$

### Hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \text{ or } (a \sec \theta, b \tan \theta)$$

$$\text{asymptotes } y = \pm \frac{b}{a}x$$

## CALCULUS

### Differentiation

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
$\ln x$	$\frac{1}{x}$
$e^{ax}$	$ae^{ax}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\cot x$	$-\operatorname{cosec}^2 x$

### Product rule

$$(f \cdot g)' = f \cdot g' + g \cdot f' \text{ or if } y = uv \text{ then } \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

### Quotient rule

$$\left(\frac{f}{g}\right)' = \frac{g \cdot f' - f \cdot g'}{g^2} \text{ or if } y = \frac{u}{v} \text{ then } \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

### Integration

$f(x)$	$\int f(x) dx$
$x^n$	$\frac{x^{n+1}}{n+1} + c$
$\frac{1}{x}$	$\ln x  + c$
$\frac{f'(x)}{f(x)}$	$\ln f(x)  + c$

### Composite Function or Chain rule

$$(f(g))' = f'(g) \cdot g'$$

$$\text{or if } y = f(u) \text{ and } u = g(x) \text{ then } \frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

### Volume of Revolution

$$y = f(x) \text{ between } x = a \text{ and } x = b$$

rotated about the x-axis.

$$\text{Volume} = \int_a^b \pi y^2 dx$$

## NUMERICAL METHODS

### Trapezium Rule

$$\int_a^b f(x) dx \approx \frac{1}{2} h [y_0 + y_n + 2(y_1 + y_2 + \dots + y_{n-1})]$$

$$\text{where } h = \frac{b-a}{n} \text{ and } y_r = f(x_r)$$

### Simpson's Rule

$$\int_a^b f(x) dx \approx \frac{1}{3} h [y_0 + y_n + 4(y_1 + y_3 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2})]$$

$$\text{where } h = \frac{b-a}{n}, y_r = f(x_r) \text{ and } n \text{ is even.}$$

## TRIGONOMETRY

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

### Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

### Cosine Rule

$$c^2 = a^2 + b^2 - 2ab \cos C$$

### Identities

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \operatorname{cosec}^2 \theta$$

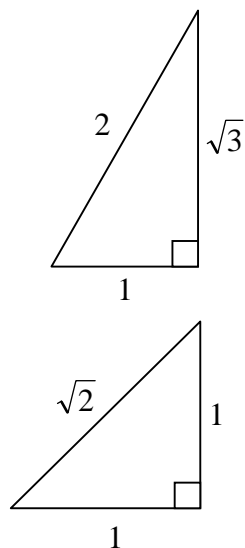
### General Solutions

$$\text{If } \sin \theta = \sin \alpha \text{ then } \theta = n\pi + (-1)^n \alpha$$

$$\text{If } \cos \theta = \cos \alpha \text{ then } \theta = 2n\pi \pm \alpha$$

$$\text{If } \tan \theta = \tan \alpha \text{ then } \theta = n\pi + \alpha$$

where  $n$  is any integer



## Compound Angles

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

## Double Angles

$$\sin 2A = 2 \sin A \cos A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2 \cos^2 A - 1 \\ &= 1 - 2 \sin^2 A \end{aligned}$$

## Products

$$2 \sin A \cos B = \sin(A + B) + \sin(A - B)$$

$$2 \cos A \sin B = \sin(A + B) - \sin(A - B)$$

$$2 \cos A \cos B = \cos(A + B) + \cos(A - B)$$

$$2 \sin A \sin B = \cos(A - B) - \cos(A + B)$$

## Sums

$$\sin C + \sin D = 2 \sin \frac{C + D}{2} \cos \frac{C - D}{2}$$

$$\sin C - \sin D = 2 \cos \frac{C + D}{2} \sin \frac{C - D}{2}$$

$$\cos C + \cos D = 2 \cos \frac{C + D}{2} \cos \frac{C - D}{2}$$

$$\cos C - \cos D = -2 \sin \frac{C + D}{2} \sin \frac{C - D}{2}$$

## MEASUREMENT

### Triangle

$$\text{area} = \frac{1}{2} ab \sin C$$

### Trapezium

$$\text{area} = \frac{1}{2} (a + b) h$$

### Sector

$$\text{area} = \frac{1}{2} r^2 \theta$$

$$\text{arc length} = r\theta$$

### Cylinder

$$\text{volume} = \pi r^2 h$$

$$\text{curved surface area} = 2\pi r h$$

### Cone

$$\text{volume} = \frac{1}{3} \pi r^2 h$$

$$\text{curved surface area} = \pi r l \text{ where } l = \text{slant height}$$

### Sphere

$$\text{volume} = \frac{4}{3} \pi r^3$$

$$\text{surface area} = 4\pi r^2$$