NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

Level 2, 2004

Mathematics

Manipulate algebraic expressions and solve equations (90284)

Sketch and interpret non-linear graphs (90285)

Find and use straightforward derivatives and integrals (90286)

Solve problems using a coordinate geometry method (90287)

Solve straightforward problems involving sequences (90290)

Solve straightforward trigonometric equations (90292)

National Statistics

Assessment Report

Assessment Schedule

Mathematics, Level 2, 2004

General Comments

Many candidates are jeopardising their chances of achieving the standard by only attempting what they perceive to be achievement questions. Often the replacement evidence is achievable in the merit questions, and the candidates should be encouraged to attempt these.

It is important for teachers to have read both the standard and the specifications for each of the standards, and ensure that the students have a clear understanding of their content.

Teachers must also be aware of the evidence required from a candidate who uses a graphics calculator. This information is again available on the NZQA website. Candidates who do not provide any working cannot receive credit where a minor error has occurred.

Candidates must understand and know what is required of them for standard mathematical terms such as solve, differentiate, expand, and they must know the meaning of terms such as, parallel, medians, altitude and the like.

Candidates must expect to get two solutions for problems involving quadratics and write coordinates in the appropriate form, giving both the x and y values.

Algebra skills such as the use of logs, manipulation of indices, writing equations, factorising, expanding, simplifying and solving equations are important in many of the standards, and lack of ability in these skills disadvantages candidates across the standards.

Candidates must be able to relate their solution back into the context of the question in order to solve problems.

More care is needed with basic numerical skills.
Far too many candidates do not attempt any questions in a booklet.
The skills required in the standard are expected to be used.
Solutions "out of the air" were common from incorrect working.
Abandoned work was often not crossed out and two answers were presented.

Mathematics: Manipulate algebraic expressions and solve equations (90284)

National Statistics

Number of Results	Percentage achieved			
	Not Achieved	Achieved	Merit	Excellence
24 597	50.1	26.9	21.5	1.5

Assessment Report

Many candidates, particularly those who used guess and check methods, only provided one answer to quadratic equations.

Candidates did not recognise the need to factorise the rational expression before simplifying and frequently cancelled incorrectly.

Questions involving logs were generally well done, although the notation used within the problem was often incorrect.

Candidates could not simplify indices correctly and the answer of $8x^3$ was common in Question 2. This meant the students thought the square root of 16 was 8.

Many did not recognise the factored form already given in 4(a) (solving a factored cubic equation) and attempted to expand and simplify the question without giving any numerical answers. This question was poorly done.

Many could not use the quadratic formula correctly, and were unable to solve the quadratic equations using factorising methods as well.

Question 5:
Many students used a Guess and Check method to get the correct answer, rather than solving the exponential equation. A common error was simplifying $500(0.75)^t = 375^t$

Question 6:
$(3x+4)^2$ was often incorrectly expanded to $9x^2 + 16$. Many other minor expansion errors occurred in this question. The poor use of [,] and (,) types of brackets when presenting coordinate answers was evident, ie the use of set notation as compared with co-ordinate notation. In Q6, the $x = 0$ was often ignored as being an irrelevant answer.

Question 7:
Brackets missing in $1600 + 18x (x – 15) = 1200 + 22x (x – 20)$ simplified the problem. In Q7, some students did not select the correct answer, although having done correct solving.

Question 8:
Students seemed to anticipate using the discriminant in this question, which was in fact a totally wrong approach.

There were many different approaches to solving Q8, and students who achieved excellence presented clear, logical, mathematically sound arguments when solving the excellence question. Methods included sums and products of roots, simultaneous equations and quadratic formula to name a few.

It was often obvious that students did not always read the questions and answer the questions asked.
Eg students seemed to ignore the $x = 0$ answer when solving a quadratic without relating the relevance of $x = 0$ as an answer to the problem.

Mathematics: Sketch and interpret non-linear graphs (90285)

National Statistics

Number of Results	**Percentage achieved**			
	Not Achieved	**Achieved**	**Merit**	**Excellence**
24 000	**39.6**	**42.6**	**14.3**	**3.5**

Assessment Report

Candidates need to clearly understand sketching and drawing graphs, which is explained in the specifications.

The candidates needed to have the key features of the graph drawn in the correct position, eg intercepts, asymptotes etc; these were often ignored or roughly sketched.

Features of graphs that were acceptable had been clearly defined in the specifications.

Candidates gave general features of the graph rather than specific features for the graph that was drawn. Statements such as, it is a circle did not provide evidence. The equation of a graph is also not considered to be a feature of the graph. As indicated in the specifications, the features needed to be three different ones ie intercepts only counted as one piece of evidence.

Evidence was accepted from question four where the candidates used features in order to answer directed questions, rather than actually describing features. Well-prepared candidates coped easily. There was usually a very clear distinction between those who could and those who couldn’t. There were very few borderline candidates.

- Some candidates with Graphics Calculators did not know how to use them effectively, ie they failed to transfer the key features from their calculator to the sketch of their graphs.
- Some candidates appeared not to know that non-linear graphs must be drawn without the use a ruler.

- Many candidates did not correctly read the scales on the axes.
- A large number of candidates did not demonstrate any evidence of understanding the nature of the features and what was required, as indicated in the specifications. The number of candidates who used correct mathematical language in describing the features was very disappointing for even basic terms, such as radius and centre of the circle.
- The omission of "y = " in giving equations was also very disappointing.
- The aspect of merit that most of the candidates had difficulty with was the sketching of the graph.
- Question 4(c) assessed the candidate's understanding of the graph and could not be answered by use of the graphics calculator alone.

1(a) Many candidates did not go much beyond plotting the intercepts when sketching the cubic. Those who did not achieve often treated it as a parabola – ignoring the common *"x"* factor.

1(b) Many candidates needed to take more care reading off the scale before sketching graphs.

1(c) Some candidates stopped sketching the log graph once it hit the x axis, and some misread the scale and had the graph going through (10,0.5) instead of (10,1); others graphed $y = x$.

1(d) Many candidates did not write down co-ordinates correctly or failed to recognise and describe the features. Many stated the obvious – it is a circle rather than specific features of the given graph. There was poor knowledge of mathematical terms associated with a circle.

2(b) More candidates appeared to use the intercept method to draw the parabola than the translation method.

3(a) The vertex of the parabola was incorrectly placed by many candidates.

3(b) More candidates had difficulties with the shape and the x intercept than expected. Many failed to recognise the negative cubic and had difficulty with the point of inflection, frequently drawing a graph of the form $y = (x - a)(x + b)(x + c) + d$ with only one x intercept.

3(c) Generally well done.

4(a) Generally well done.

4(b) Most common mistake was giving an answer of 10, which indicates many candidates did not read and interpret the question correctly. Other candidates gave the exact value indicating again lack of interpretation of the question.

4(c) Most candidates found that communicating the mathematical ideas about how the graph was affected a challenge. Some used a sketch on the graph to assist, and these candidates generally responded better.

5 Most candidates did not attempt this question, including some who gained Merit with evidence to spare. Those attempting the question usually made a good attempt and usually had success. Some managed to substitute values in the formula and very few went on to eliminate one of the variables, perhaps because logs were involved.

Mathematics: Find and use straightforward derivatives and integrals (90286)

National Statistics

Number of Results	Percentage achieved			
	Not Achieved	Achieved	Merit	Excellence
23 380	59.9	29.8	7.7	2.5

Assessment Report

- Candidates tended to be stronger in integration than differentiation, and evidence is required that the candidate has the ability to perform both skills.
- Candidates must understand that they are required to state the integrated or derived function as well as using this.

- Understanding of the relationship between integral and area under a curve, and derivative and gradient, is important as is the candidate's understanding of the different calculus notations.
- Candidates often omitted the constant of integration.
- Selection of correct limits for integration was a problem for some candidates.
- Candidates need to be clear in their understanding of the relationship between distance, velocity and acceleration.
- Candidates must be able to form their own equations from information given and reduce the initial equation to one with only one variable.

Mathematics: Solve problems using a coordinate geometry method (90287)

National Statistics

Number of Results	Percentage achieved			
	Not Achieved	Achieved	Merit	Excellence
24 992	35.1	38.4	16.0	10.6

Assessment Report

There was a pleasing improvement in the knowledge of the formulae from the previous year.

Poorer candidates tried to work from formulae rather than understanding, and were generally unable to recollect even the most basic formulae such as the midpoint. Confusion between the midpoint and the gradient formula was common.

The distance formula also provided problems, and the connection needs to be emphasised on the relationship between this formula and Pythagoras to help understanding of this concept.

Question 4 involved a proof and provided most candidates, including some very good ones, with problems. The techniques involved in tackling and presenting a proof is something that teachers should spend more time on in the future, as the coordinate geometry topic does provide many opportunities for proofs to be assessed.

Question 5, the Excellence question, was very logically set out and well tackled by the successful candidates, perhaps reflecting the similarity with the previous year's question.

Some candidates let themselves down on these questions by assuming an isosceles triangle or a right angle existed in the problem. This needed to be shown in their working if they were to get credit for this.

Mathematics: Solve straightforward problems involving sequences (90290)

National Statistics

Number of Results	Percentage achieved			
	Not Achieved	Achieved	Merit	Excellence
20 914	19.0	56.2	19.0	5.9

Assessment Report

Although students generally identified correctly the type of sequence to be used for each question, candidates assessed as Not Achieved did not recognise the type of sequence involved and therefore did not choose the appropriate formula.

Candidates gaining Merit and Excellence understood the sum to infinity concept and had a better grasp of percentage changes.

A significant proportion of students did not achieve credits in this achievement standard because they did not use the correct order of operations. Students should be encouraged to check the reasonableness of their answers in the context of the question.

Mathematics: Solve straightforward trigonometric equations (90292)

National Statistics

Number of	Percentage achieved			
Results	Not Achieved	Achieved	Merit	Excellence
20 369	38.5	29.2	26.2	6.1

Assessment Report

Candidates who did not achieve did not recognise the periodic nature of trigonometric equations, which give rise to multiple solutions.

Some candidates tried to use general solution formulae without the knowledge required to correctly interpret them.

Students who achieved showed the ability to draw accurate sketch graphs of trigonometric functions, which incorporated one or more transformations, and then interpret their symmetry.

Whilst candidates were not penalised ***this year*** for giving answers in the units other than specified in the domain, the ability to work in both degrees and radians needs to be demonstrated, and thence correctly written answers with units matching the domain stated.

Some candidates rounded inappropriately, especially premature rounding to 1 or 2 significant figures.

Assessment Schedule
Mathematics: Manipulate algebraic expressions and solve equations (90284)
FUN AND GAMES WITH ALGEBRA

	Assessment Criteria	No.	Evidence	Code	Judgement	Sufficiency
Achievement	Manipulate algebraic expressions and solve equations.	1	$x-2$	A1	Or equivalent.	**Achievement:** Part 1: Algebraic expressions Two of Code A1 No skills repeated **AND** Part 2: Equations: Two of Code A2
		2	$4x^3$	A1	Or equivalent.	
		3	log 6	A1	Or equivalent.	
		4(a)	$x = 0, 4, -0.5$	A2	All answers required.	
		4(b)	$x = 4$	A2	No alternative.	
		4(c)	$x = -2, 1.5$	A2	Both answers required.	
Merit	Solve problems involving equations.	5	$0.5 = (0.75)^t$ $t\log(0.75) = \log 0.5$ $t = 2.409$ hours or 2 hrs 25 min	 A1 A2 M	Units not necessary. Accept any rounding	**Merit:** Achievement plus two of Code M **OR** all of Code M.
		6	$10x^2 + 24x = 0$ 0 and – 2.4	A1 A2 M	Must have both answers.	
		7	$(1600 + 18x)(x - 15)$ $= (1200 + 22x)(x - 20)$ $4x^2 - 570x = 0$ $2x(2x - 285) = 0$ $x = 0, x = 142.5$ need 143 people	 A1 A2 M	Students must identify the equation that is to be solved as well as the solution. Must choose correct answer of the two Accept any correct rounding.	
Excellence	Choose algebraic techniques and strategies to solve a problem.	8	$4x^2 - mx + 5 = 0$ Roots: $\frac{m+\sqrt{m^2-80}}{8} = 3\left(\frac{m-\sqrt{m^2-80}}{8}\right)$ $4\sqrt{m^2-80} = 2m$ $3m^2 - 320 = 0$ $m^2 = 106\frac{2}{3}$ $m = \pm 10.33$ (2dp) or $m = \pm 16\sqrt{\frac{5}{12}}$	 A1 A2 M E	Must have clear explanation of the process being used to solve the problem. Need both answers. **Or** equivalent. Any correct rounding.	**Excellence:** Merit plus Code E.

Assessment Schedule
Mathematics: Sketch and interpret non-linear graphs (90285)

	Assessment Criteria	No.	Evidence	C	Judgement	Sufficiency
Achievement	Sketch non-linear graphs from equations and identify relevant features of graphs.	1(a)	Cubic – x intercepts 0, 3, and –2 y intercept 0.	G	Must pass through intercepts. TPs do not need to be accurate. Smooth cubic curve required.	2 Code G **and** 2 Code I.
		1(b)	Sine graph drawn centred at $y = 2$.	G	Smooth curve through (0,2) (90,3) (180,2) (270,1) (360,2) required. Accept through (270,1.5)	
		1(c)	Correctly shaped log graph through (1,0) and approaching (10,1).	G	The graph must not cross the y-axis or significantly bend back. Smooth curves required.	
		1(d)	1. Centre (0,–4) 2. Radius 5 3. x intercepts –3,3 **OR** y intercepts –9 and 1 **OR** range 1,–9 4. Domain (–5,5)	I I I		
Merit	Write equations of graphs.	2(a)	$y = \frac{4}{x}$ or $xy = 4$	Eq	Accept 4 – 3.5	**Achieve** plus 2 Code MG **and** 2 Code MI **and** 2 Code Eq. **OR** 3 Code MG **and** 3 Code MI **and** 2 Code Eq.
		2(b)	$y = (x-3)^2 - 1$ or $y = (x-2)(x-4)$	Eq	$y = x^2 - 6x + 8$	
		2(c)	$y = 3^x$ or $x = \log_3 y$	Eq		
	Plot graphs of equations and	3(a)	Parabola x intercepts –1,1.5 y intercept –3 Minimum (0.25,–3.125) Axis of symmetry $x = 0.25$.	MG	Smooth graph with intercepts correct and vertex x between 0 and 0.5 and y between –3 and –3.5. Intercepts need to be correct. Smooth curve. Vertex below –3 and above –4.	
		3(b)	–ve cubic y intercept 8 x intercept 2.	MG	Smooth curve through (0,8) and close to (2,0) if extended.	
	interpret their features.	3(c)	exponential y intercept = 3 through (1,6) or 1 other exact point, x-axis as asymptote.	MG	Does not touch the x-axis.	
		4(a)	\$16 000.	MI	No alternative.	
		4(b)	11 years.	MI	Or consistent with (a) (15 000 → 10 years)	
		4(c)	At $x = 5$ the graph drops \$2000 and then the graph is flatter.	MI	Generally stated drop and either quantified by \$2000 or comment on flattening of gradient. May be shown by diagram. *(Any 1 of these is evidence for I.)*	

Excellence	Determine and apply an appropriate graphical model for a situation.	5(a)	Equation $8 = A\log 3 + B$ $18 = A\log 6 + B$ $F = 33.21928 \log(x + 2) - 7.8496$ $F = 33.22 \log(x+2) - 7.85.$	I Eq or E	 Equivalent equations acceptable or a correct statement of A and B. Consistent. Ignore rounding error.	**Merit** plus 2 Code E.
		5(b)	At the end of 21 days the account will have $37 386.59 or $37 387.	E	Accept 37 000	

Assessment Schedule
Mathematics: Find and use straightforward derivatives and integrals (90286)

	Assessment Criteria	No.	Evidence	Code	Judgement	Sufficiency
Achievement	Find and use straightforward derivatives and integrals.				Units not required anywhere in this task.	
		1(a)	$\frac{dy}{dx} = 8x^3 - 1$ when $x = 1$, gradient = 7.	A1	Both derivative and value are required.	**3 of Code A** including at least **1 of each of A1 and A2.**
		1(b)	$y = 2x^4 + x^2 - 4x + c$ $c = -13$ $y = 2x^4 + x^2 - 4x - 13.$	A2	Both integral (anti-derivative) and c required. No alternative. Indicate omission, then ignore lack of $y =$, $f(x) =$ Accept integration $+c$, then correct calculation of c, without final rewriting of equation.	
		1(c)	Area $= \left\|\int_0^2 x^2 + 4\right\|$ $= \left\|\left[\frac{x^3}{3} + 4x + c\right]_0^2\right\|$ $= 10\frac{2}{3}.$	A2	Both integral and area required. Accept substitution of 0 before 2, provided final answer is positive.	
		(d)	$\frac{dy}{dx} = -8x^{-3}$ $= 1$ $x = -2$ $y = 1.$	A1	Derivative and both x and y values are required.	

	Assessment Criteria	No.	Evidence	Code	Judgement	Sufficiency
Merit	Apply calculus techniques to solve problems.	2	Area $= \left\lvert \int_0^2 (x^2 - x - 2)\,dx \right\rvert + \left\lvert \int_2^4 (x^2 - x - 2)\,dx \right\rvert$ $= \left\lvert \left[\frac{x^3}{3} - \frac{x^2}{2} - 2x + c \right]_0^2 \right\rvert + \left\lvert \left[\frac{x^3}{3} - \frac{x^2}{2} - 2x + c \right]_2^4 \right\rvert$ $= 3.\dot{3} + 8.\dot{6}$ $= 12.$	M (A2)	Units not required in any question. Need integration shown. Equivalent approaches allowed, provided a progression to final answer is evident. No Alternative. A2 for correct integration and calculation of either shaded part, or integral from 0 to 4.	**Achievement** plus **2 Code M** OR **3 Code M.**
		3(a)	$s(t) = \int (60 - 3t^2)\,dt$ $= 60t - t^3 + c$ $s(10) = 600 - 1000 + c = 15$ $c = 415$ (cm).	M (A2)	A candidate who acknowledges contradiction with domain gains M for 3(a). Ignore omission of +c. No Alternative.	
		3(b)	$\frac{dV}{dt} = -6t$ $= 0$ For turning point $\Rightarrow t = 0$ $\Rightarrow v = 60$ (cm per sec).	M (A1)	No Alternative. A1 for derivative and solving $t = 0$. Use of calculus must be evident.	

	Assessment Criteria	No.	Evidence	Code	Judgement	Sufficiency
Excellence	Apply differentiation techniques to solve optimisation problems.	4	$\pi r^2 h = 10$ $h = \dfrac{10}{\pi r^2}$ $A = \dfrac{10}{\pi r^2}.2\pi r + 2\pi r^2$ $= \dfrac{20}{r} + 2\pi r^2$ $\dfrac{dA}{dr} = 4\pi r - \dfrac{20}{r^2} = 0$ $4\pi r^3 = 20$ $r^3 = \dfrac{20}{4\pi}$ $r = \sqrt[3]{\dfrac{20}{4\pi}}$ or $\sqrt[3]{\dfrac{5}{\pi}}$ For a volume of 10 cm^3 with minimum surface area radius $= \sqrt[3]{\dfrac{20}{4\pi}} = 1.1675\ (4\text{dp})$.	E (M) (A1)	– Rearrange – Substitute – Differentiate – Solve Accept any rounding including premature rounding. No alternative. M or A1 for taking derivative and finding *r*.	**Merit** plus **Code E.**

Assessment Schedule
Mathematics: Solve problems using a coordinate geometry method (90287)

	Assessment Criteria	No.	Evidence	Code	Judgement	Sufficiency
Achievement	Solve problems using a coordinate geometry method.	1(a) 1(b) 1(c)	(4,12) $x - 2y + 5 = 0$ $7x + y - 80 = 0$	A A A	Or equivalent. Or equivalent. Or equivalent.	**Achievement:** 2 of A **Replacement Evidence**: Any part of Q 2, 3 or 5 can replace 1(a), (b) or (c). No repeated skills.
Merit	Solve problems involving a combination of at least two coordinate geometry methods.	2 3 4	Mpt: $R_3(13,9)$. Distance: $\sqrt{40}$. Intersection: E (6,–2). Eqn of line: $x + y - 4 = 0$. Indication $-2 \times \frac{1}{2} = -1$ gives rt angle. Show subst. (3,19) into $2x + y - 25 = 0$ or derive eqn. from $y - y_1 = m(x - x_1)$	A A M A A M M	Or equivalent. Rounding and units not required. Or equivalent. Both required.	**Achievement with Merit:** Achievement plus 2 of M or 3 × M **Replacement Evidence:** Two consecutive coordinate geometry techniques in Question 5 can replace any part of Q 2, 3 and 4.
Excellence	Choose and apply a variety of coordinate geometry methods to solve problems.	5	Finding the perp. line through *D*: $2x + y - 25 = 0$. Intersection: (9,7). Distance = $\sqrt{20}$.	A A A M E	Alternative methods acceptable. Accept a minor error in working. Solution should be logically set out and methods able to be followed.	**Achievement with Excellence:** Merit plus E.

Assessment Schedule
Mathematics: Solve straightforward problems involving sequences (90290)

SURFING THE NET

	Assessment Criteria	No.	Evidence	Code	Judgement	Sufficiency
Achievement	Solve straightforward problems involving sequences.	1(a) 1(b) 2	$a = 21$ $d = 3$ $t_{20} = 78$ min $S_{20} = 990$ min or 16 hours 30 min or 16.5 hours $S_{21} = 897.3$ min or 14 hours 57 min	A A A	Units not required anywhere in this activity.	**Achievement:** two of Code A.
Merit	Solve problems involving sequences.	3 4 5	Geometric Sequence $r^2 = 0.81$ $r = \pm 0.9$ r must be 0.9. $a = \$1\ 000$. $480 = 4\ 800\ (1 - 0.32)^n$ $0.1 = 0.68^n$ $n = 5.97$ ie 6 (years). $r = 0.8$ $S_\infty = \$375$.	M A M A M A	 Accept 6 only.	**Merit:** Achievement plus two of Code M OR all of Code M.
Excellence	Explore situations and interpret the results of problems involving sequences.	6	Arithmetic Sequence $t_9 = 1\ 490$ $S_{24} = 29\ 880$ $1\ 490 = a + 8d$ $29\ 880 = 12(2a + 23d)$ $(29\ 880 = 24a + 276d)$ $d = -70$ $a = 2\ 050$ minutes	M E	Must have supporting working.	**Excellence:** Merit plus Code E.

Assessment Schedule

Mathematics: Solve straightforward trigonometric equations (90292)

	Assessment Criteria	No.	Evidence	Code	Judgement	Sufficiency
Achievement	Solve straight-forward trigonometric equations.	1(a)	$\theta = 78.5°, 281.5°$ [1.37, 4.91]	**A**	Both required.	**Achievement:** 2 of Code **A.** **Replacement Evidence:** Any **A/M/E** from Q 2, 3, or 4 can replace evidence for any one of 1(a), (b) or (c). (no repeated skills)
		1(b)	$\theta = 36.9°, 143.1°$ [0.64, 2.50]	**A**	Both required.	
		1(c)	$\theta = 1.19, 4.33$ [68.2°, 248.1°]	**A**	Both required. If in degrees, ° needs to be shown. Accept any rounding throughout. Units not required. Degrees or radians acceptable throughout. Extra correct answers outside the specified domain could be ignored in holistic judgement.	
Achievement with Merit	Solve trigonometric equations.	2(a)	$\theta = 68.6°, 201.4°$ [1.20, 3.52]	**A M**	Both required.	**Achievement with Merit:** **Achievement** plus 2 of Code **M** **OR** all **three** of Code **M**. **Replacement Evidence:** Q 4 **M/E** for 2 or 3.
		2(b)	$\theta = 0.388, 1.183$ [22.2°, 67.8°]	**A M**	Both required. If in degrees, ° needs to be shown. Q2 (a), (b) Degrees or radians acceptable. Extra correct answers outside the specified domain could be ignored in holistic judgement.	
		3	$t = 1.14$	**A M**	Accept any rounding throughout. Units not required.	
Achievement with Excellence	Solve multi-step trigonometric problems.	4	$\cos\frac{\pi t}{6} = -0.72$		Alternative methods acceptable. Accept a minor error in working.	**Achievement with Excellence:** **Merit** plus Code **E.**
			$\frac{\pi t}{6} = 2.375, 3.909$	**A**	Solution should be logically set out and methods able to be followed.	
			$t = 4.535, 7.465.$	**M**	CAO for t and correct interpretation for time is sufficient. (See Graphic Calculator guidenotes.)	
			Hence 1:02 pm to 3:58 pm. [1pm to 4pm, 1300 to 1600]	**E**		